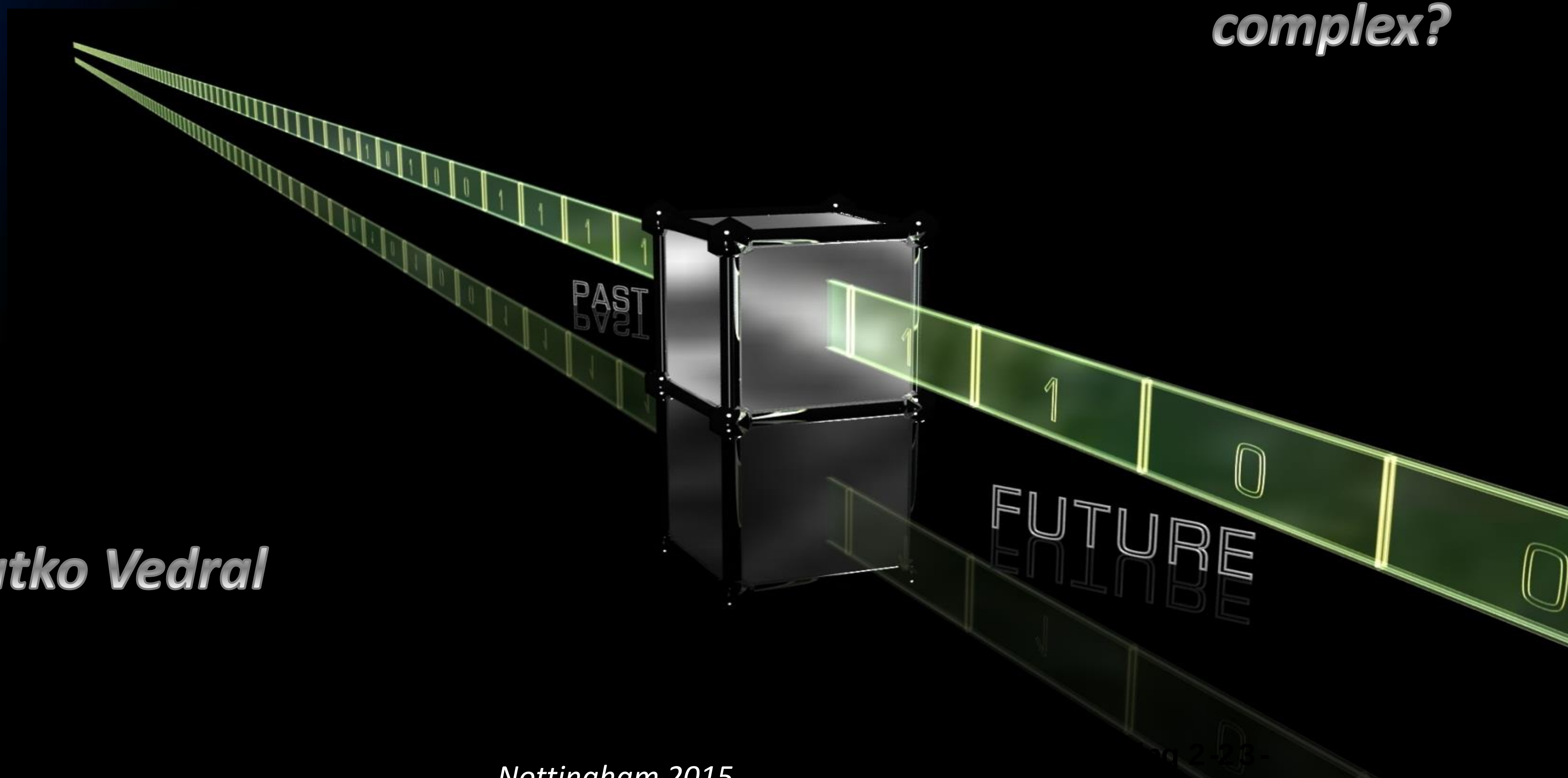
Viewing Complexity through The Quantum Lens

Can Quantum Logic reshape our notions of what is complex?



Vlatko Vedral

Nottingham 2015

"Why does "complexity" or "interestingness" of physical systems seem to increase with time and then hit a maximum and decrease, in contrast to the entropy, which of course increases monotonically?"

-Sean Carol, 2011 FQXi Conference

T = 1s

Smooth Plasma – Simple


T ~ Now
‘Very Interesting’ Patterns, Structure, Complexity

Low Entropy

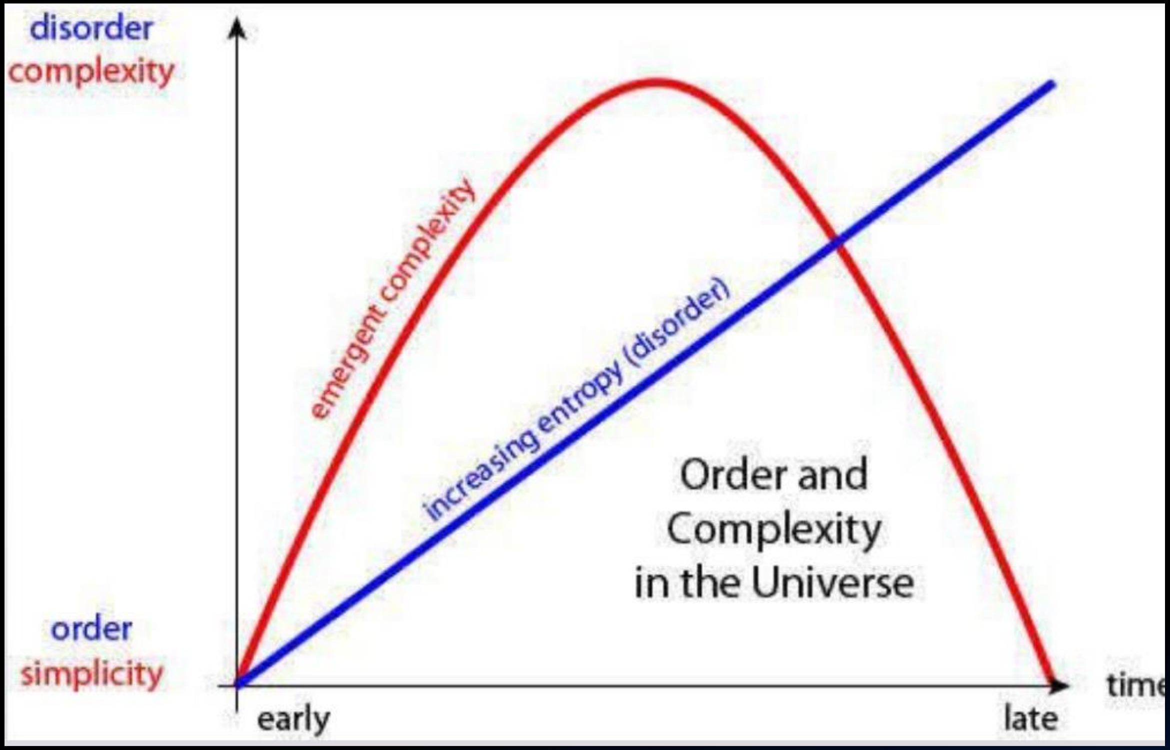
Low Complexity

High Entropy

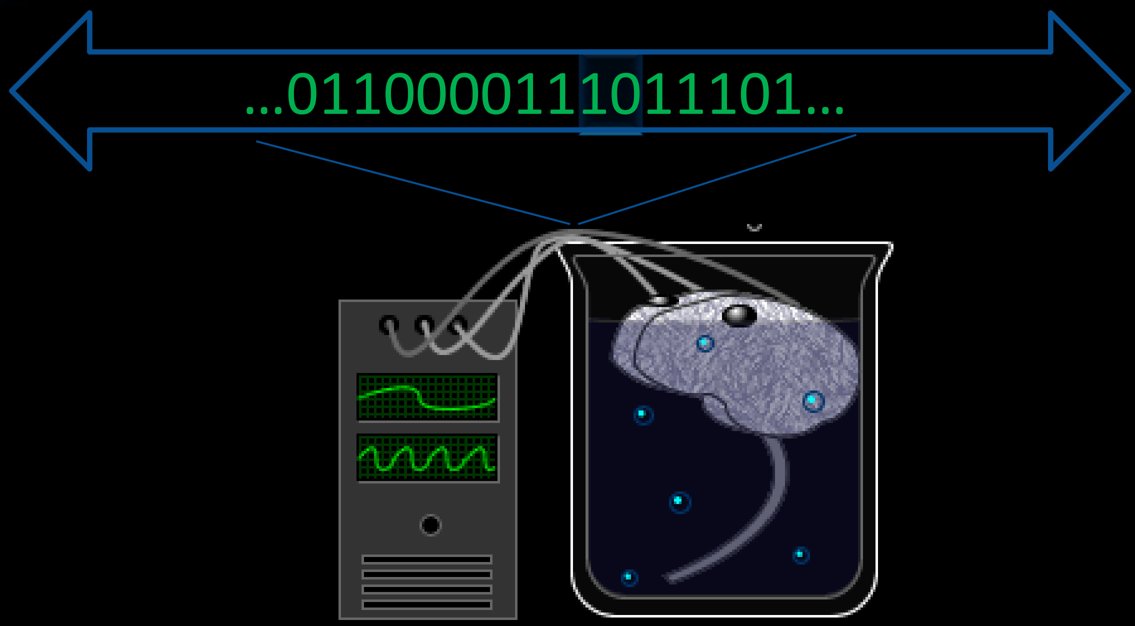
Low Complexity

High Complexity!

Time


disorder
complexity
order
simplicity
emergent complexity
increasing entropy (disorder)
Order and
Complexity
in the Universe
early
late
time

Is there a Universal Law for the Evolution of Complexity?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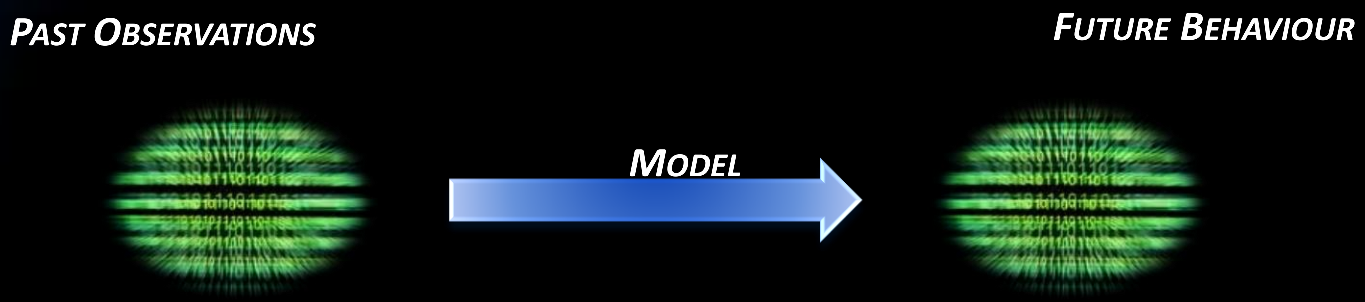
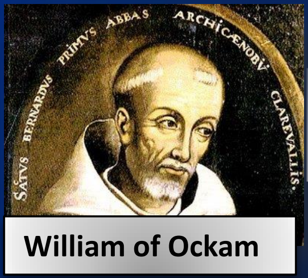
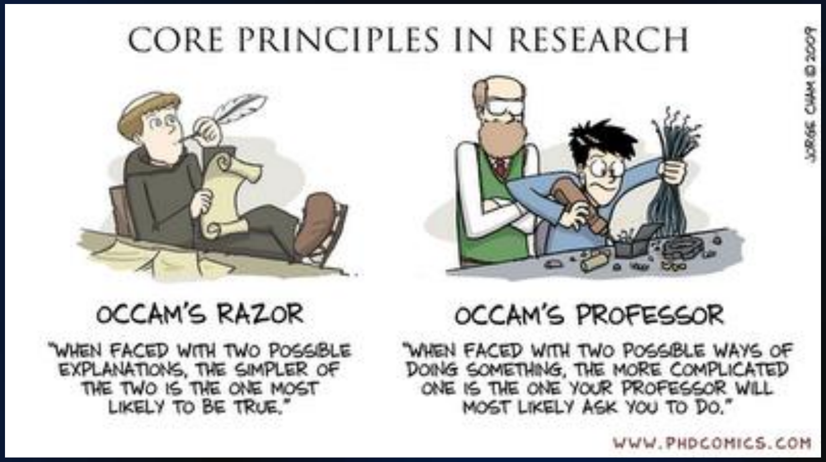

Past Observations
Model
Future Behaviour

Occam's Razor

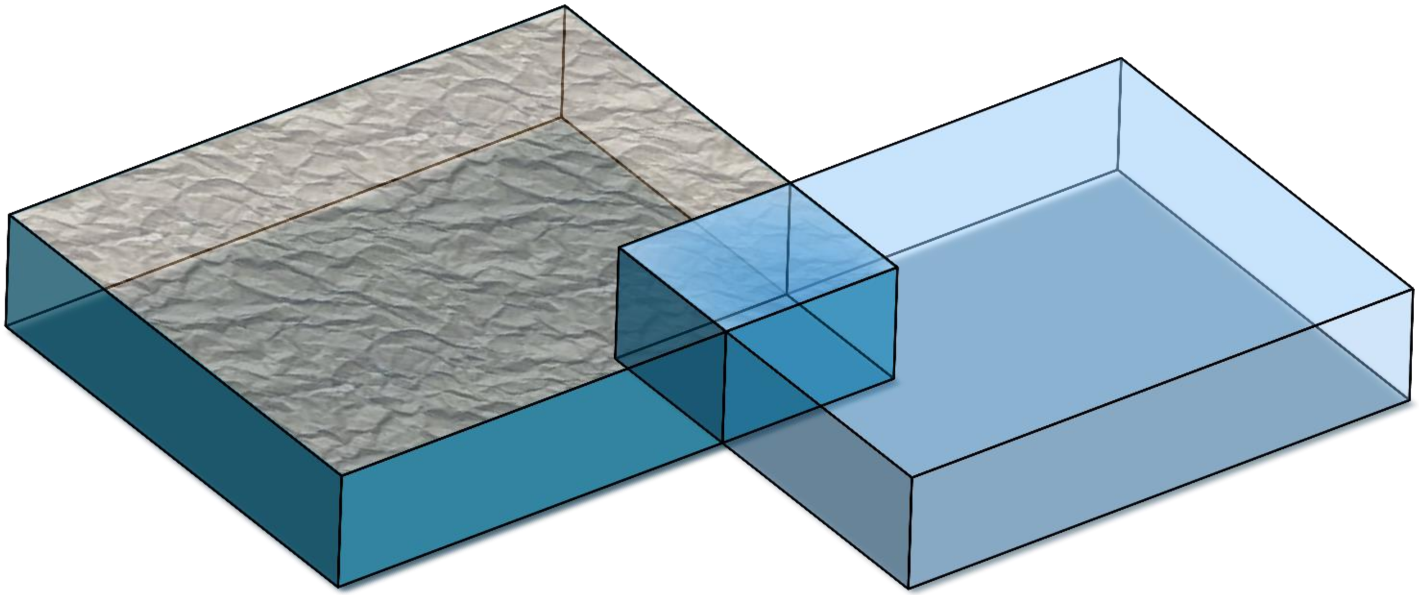


"Plurality is not to be posited without necessity."

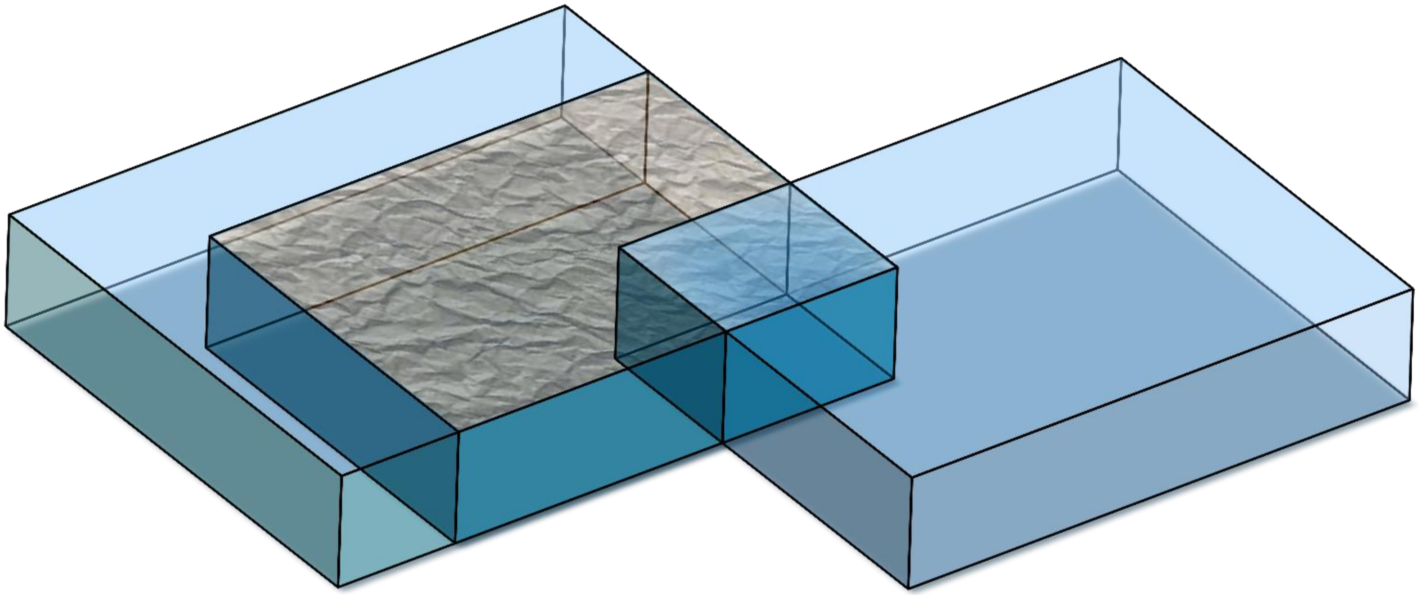
"We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances."



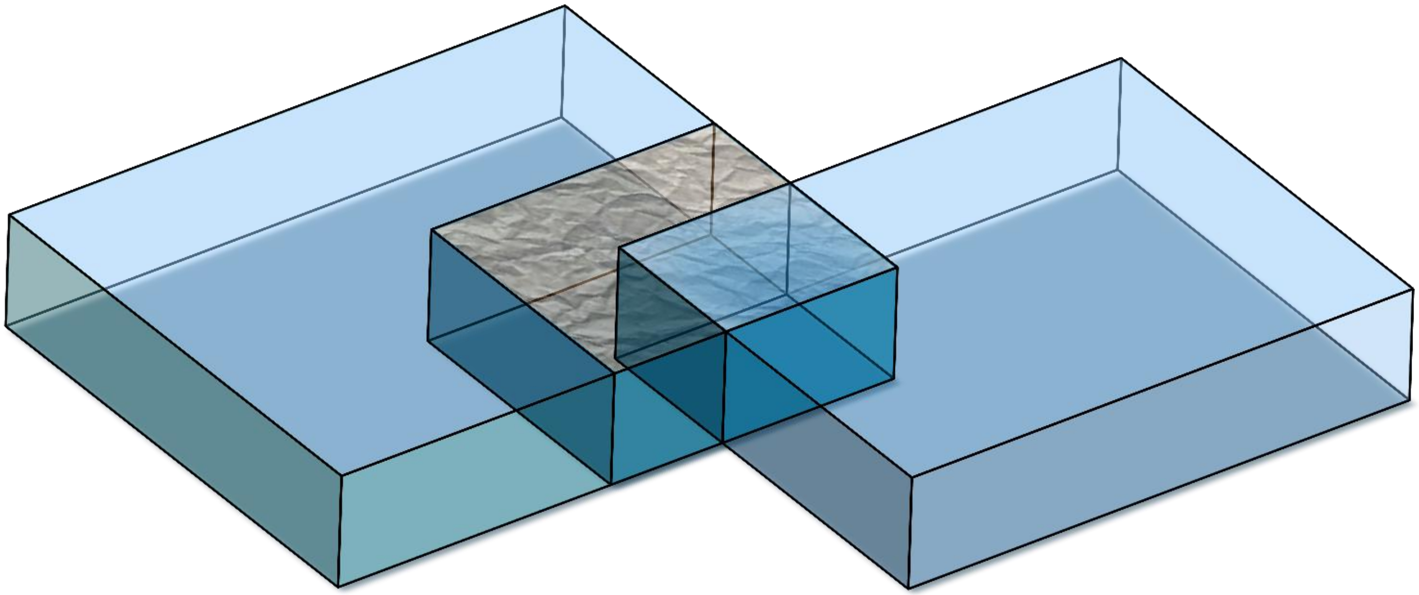
Without any understanding, every possible past is a potential cause of future events.

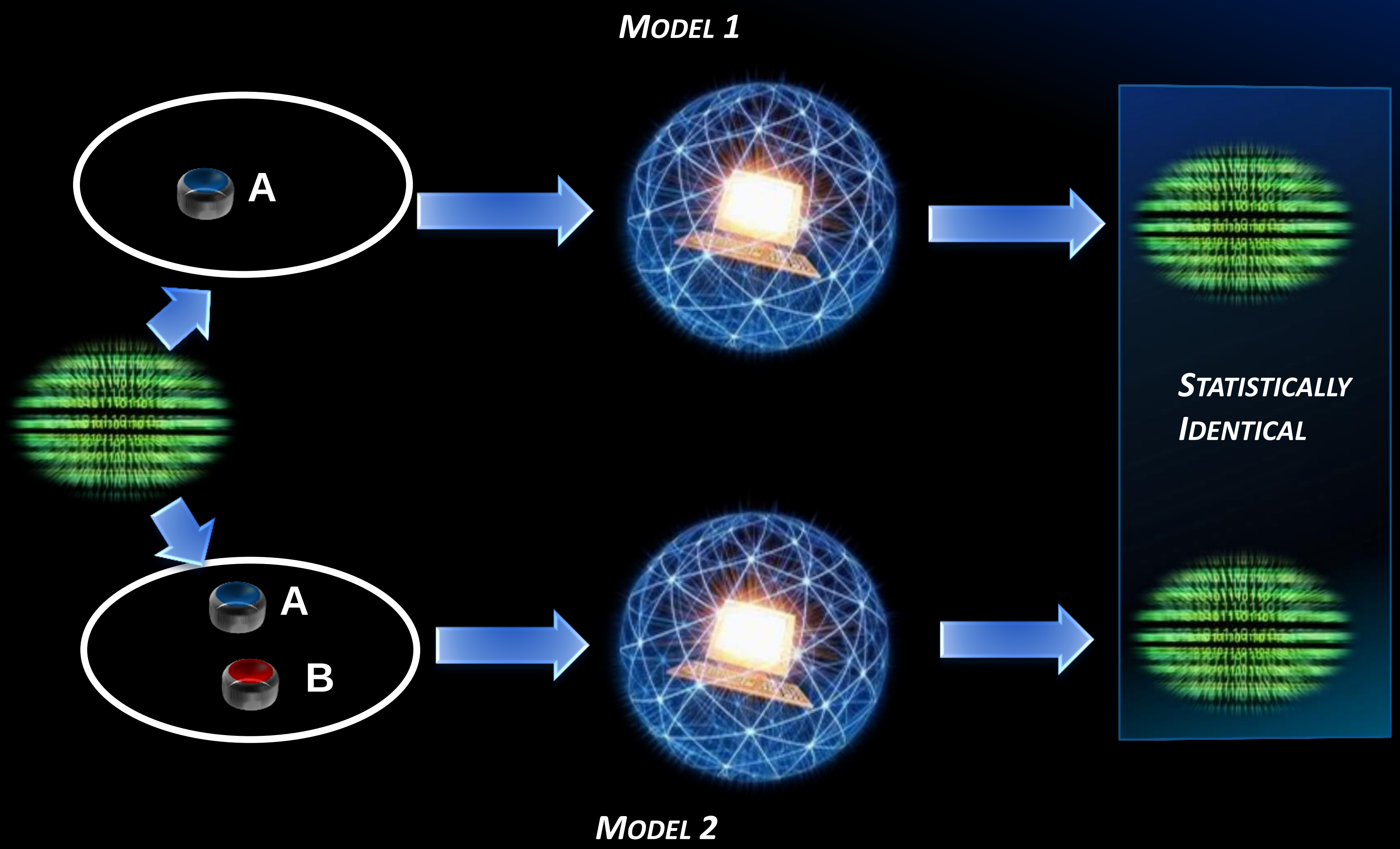


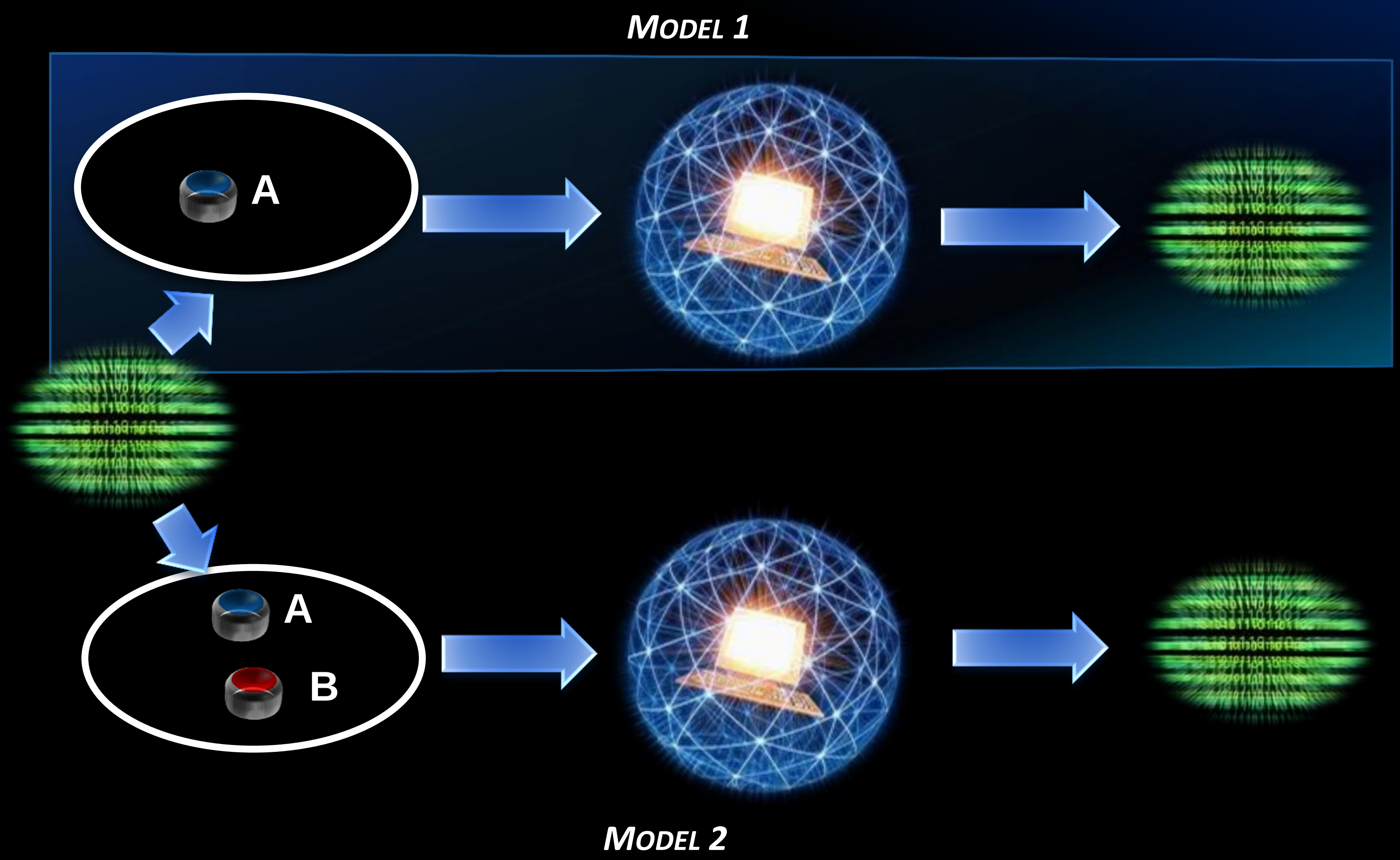
$$P(\overleftarrow{X}, \overrightarrow{X})$$

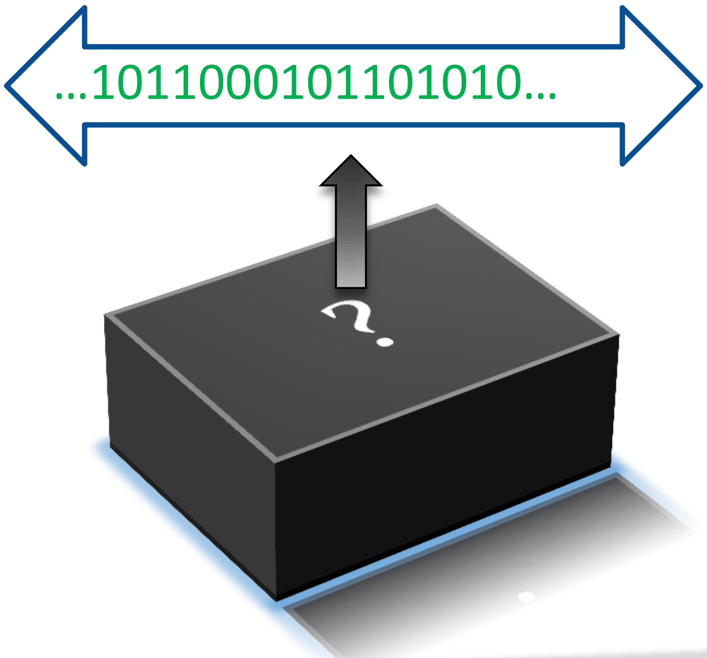


The better we can isolate the causes of natural things, the greater our understanding.



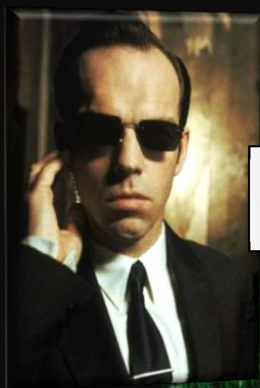

MODEL 1
A
A
B
STATISTICALLY IDENTICAL
MODEL 2

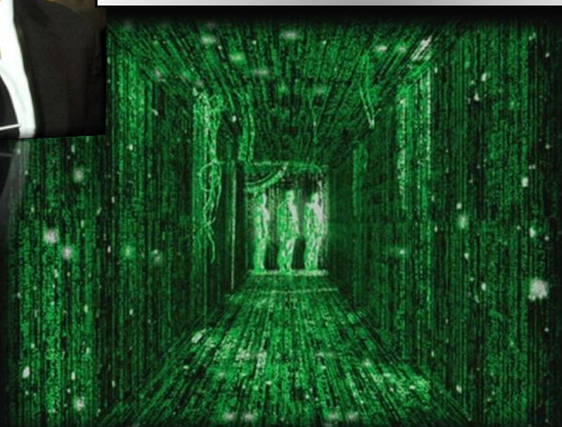

MODEL 1
A
A
B
MODEL 2



The complexity of a system can be quantified by the minimum amount of causes (as measured by information entropy) one must invoke to understand.

Enter the Matrix



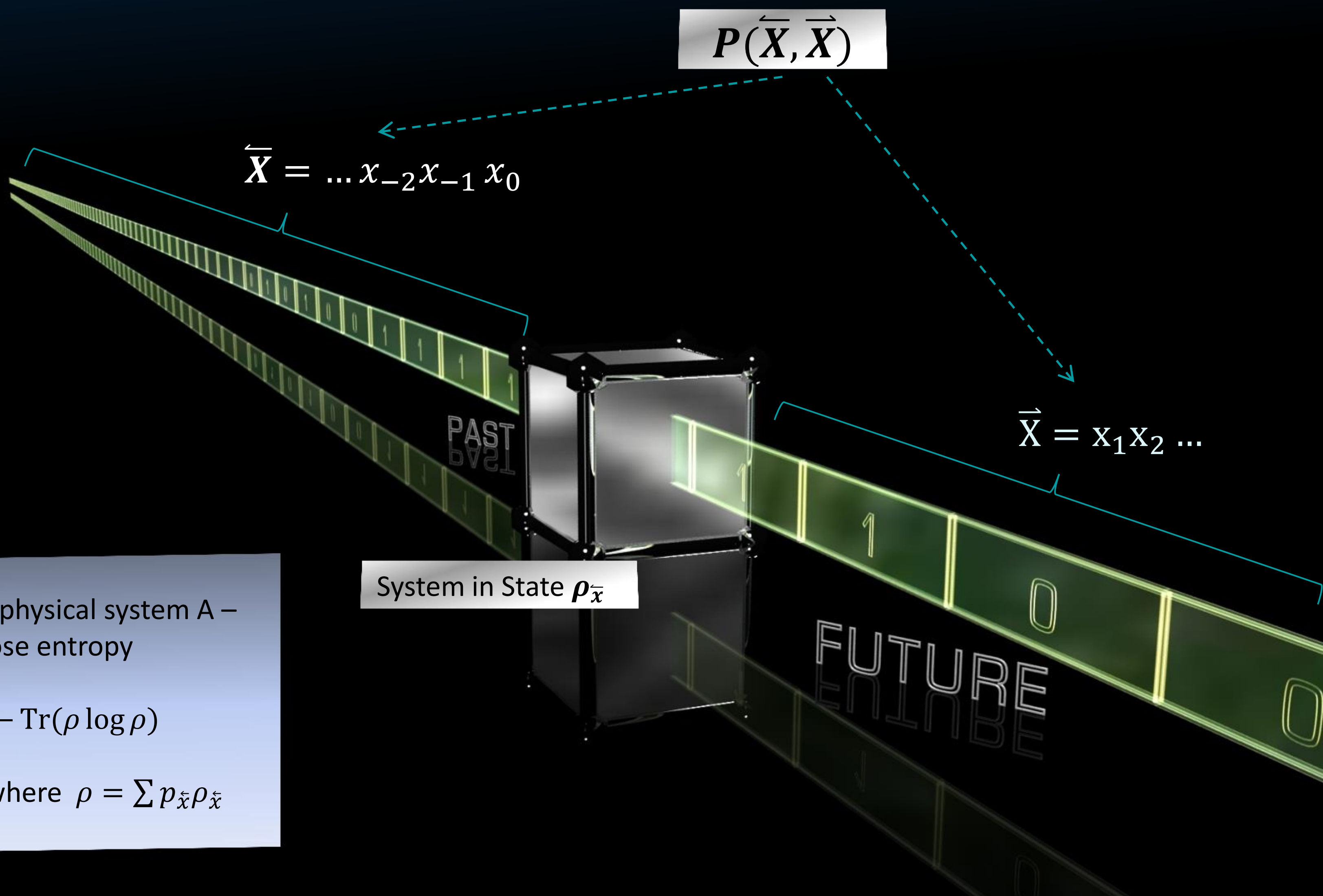


Suppose you're a programmer for the matrix

You are tasked to program an object to simulate a particular desired behaviour.

The more complex object, the more 'causes' you must track – the more hard drive space you will need.

Stochastic Processes



Task:

Find a simplest physical system A – the system whose entropy

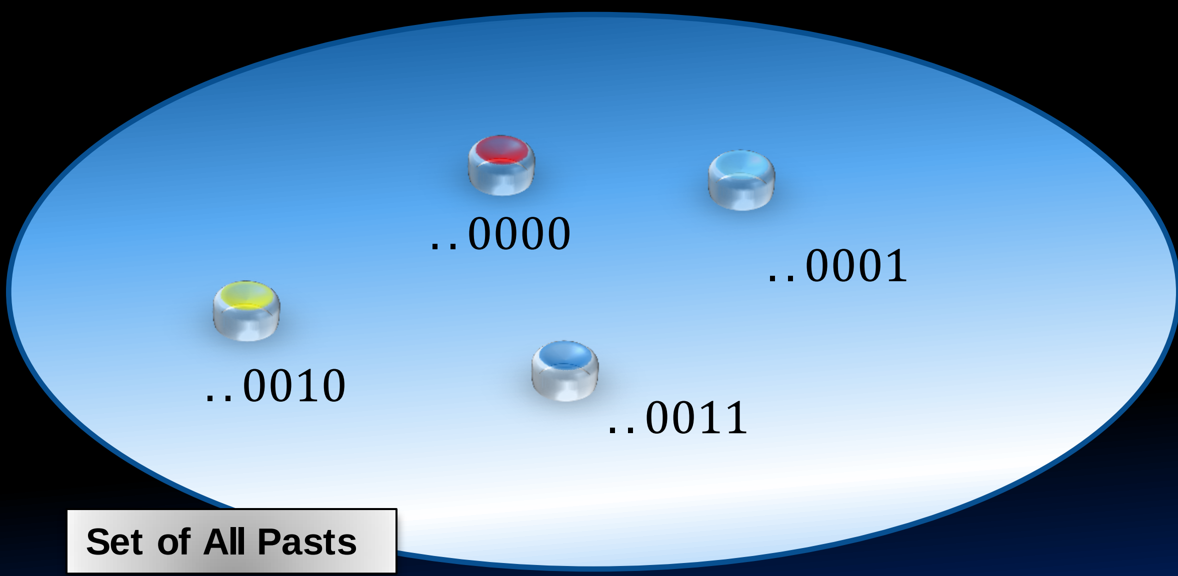
$$S(\boldsymbol{A}) = -\,\mathrm{Tr}(\rho \log \rho)$$

Is minimised, where $\rho = \sum p_{\overleftarrow{x}}\rho_{\overleftarrow{x}}$

Stochastic Processes

Construct a system that stores each possible past in a separate configuration.

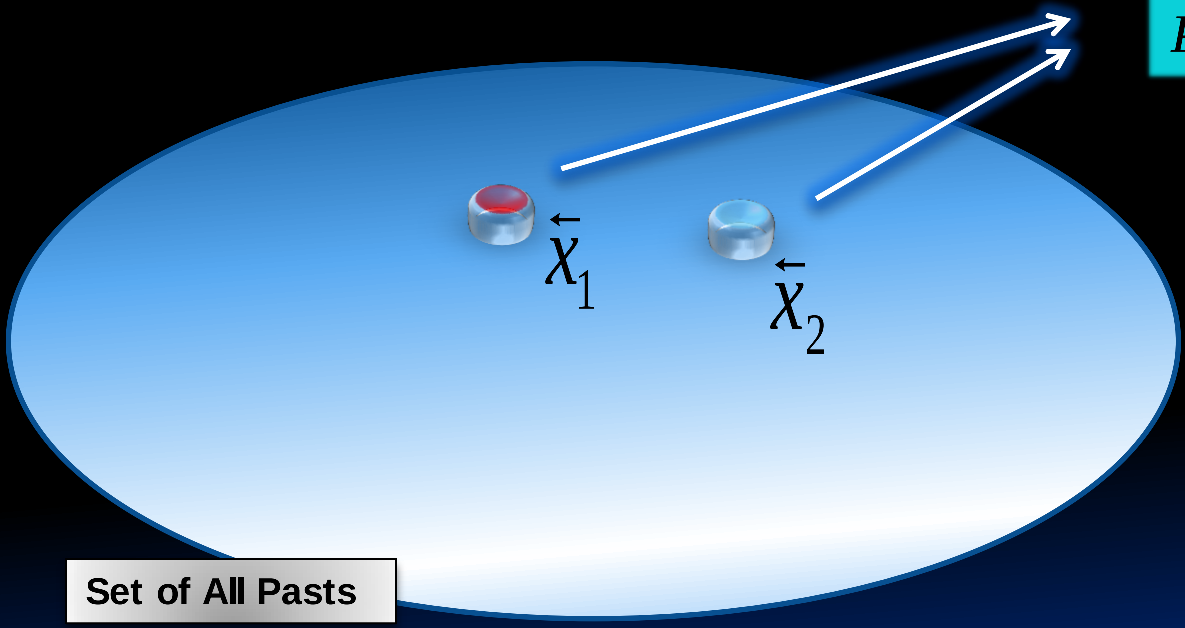
$$\rho_{\overleftarrow{x}} = \overleftarrow{x}$$



Stochastic Processes

Suppose two pasts have statistically identical futures

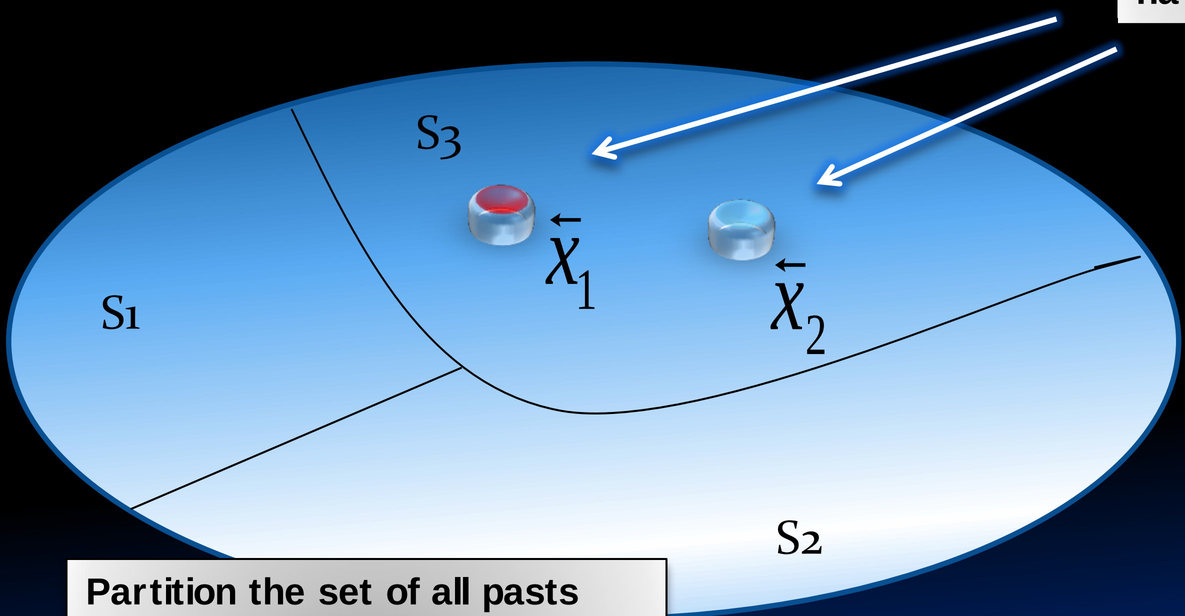
$$P(\overrightarrow{X} \mid \overleftarrow{X} = \overleftarrow{x}_1) = P(\overrightarrow{X} \mid \overleftarrow{X} = \overleftarrow{x}_2)$$



The information needed to distinguish the two is irrelevant to the future of the process and can thus be discarded.

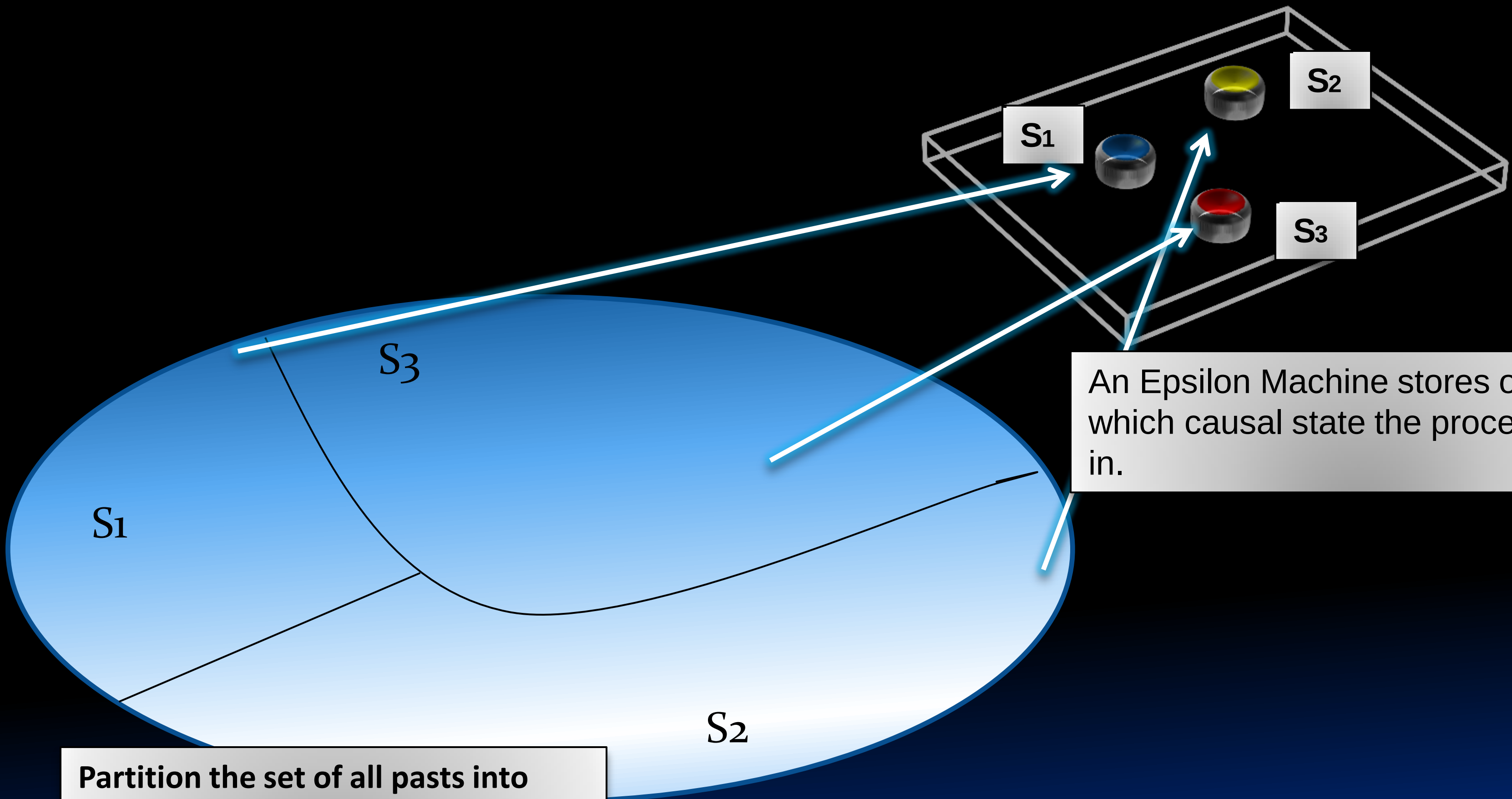
Focus on Stochastic Processes

Two different possible pasts belong to the same Causal state if they have coinciding futures.



Partition the set of all pasts into equivalence classes, referred to as Causal states.

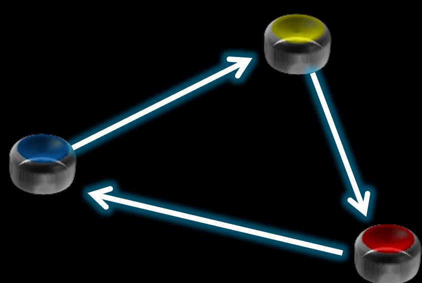
Focus on Stochastic Processes

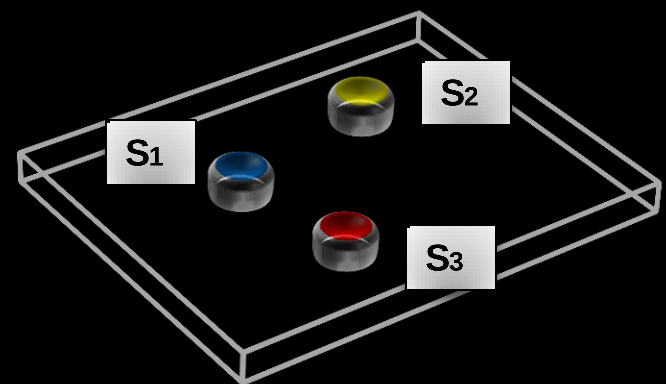


Partition the set of all pasts into equivalence classes, referred to as Causal states.

An Epsilon Machine stores only which causal state the process is in.

The Simplest Model... Epsilon Machines





An Epsilon Machine stores only which causal state the process is in.

The Stochastic Process can then be completely defined by transition probabilities on the causal states.

$$T^{r}_{j,k}$$

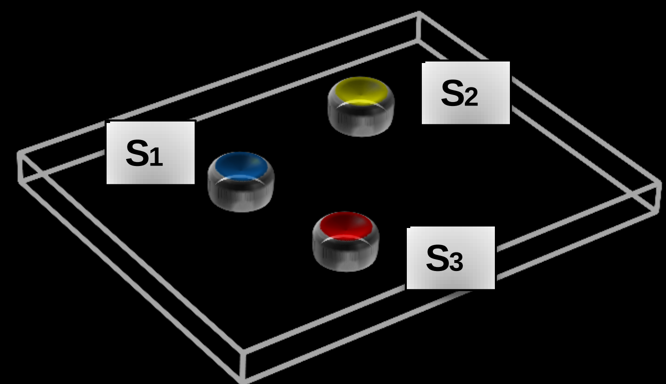
Probability a Stochastic process in Causal state S_j will emit output `r' and transition to S_k

Statistical Complexity

Probability the process is in Causal State Si

Internal Entropy:

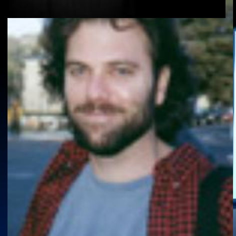
$$C_\mu = -\sum p_i \log p_i$$



To simulate a sequence of random coin flips….

We have a process with exactly 1 Causal State

No Information about the Past is required!

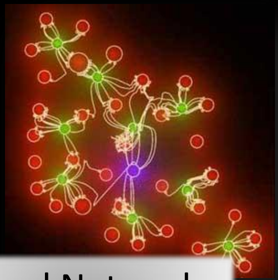


No classical system can simulate a given stochastic process using less information than a Epsilon Machine

Crutchfield, J, and Young, K. Physical Review Letters 63.2 (1989):

C_μ Is a **intrinsic** property of a stochastic process that measures the minimal amount of memory required to simulate the given process.

Statistical Complexity



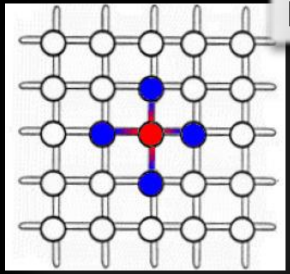
IEEE Trans. Neural Networks, 10, 2, 284-302

Neural Networks

Dripping Faucets

Physica A: 257, 1-4, 385-389

Applied to wide range of systems.



Ising Models

PRA 238, 4-5, 244-252

Pseudo-random Number generators.

Physica A: 356, 1, 133-138

Crutchfield

1989

No classical system can simulate a given stochastic process using less information than a Epsilon Machine

Phys. Rev. Lett. 63, 105–108 (1989)

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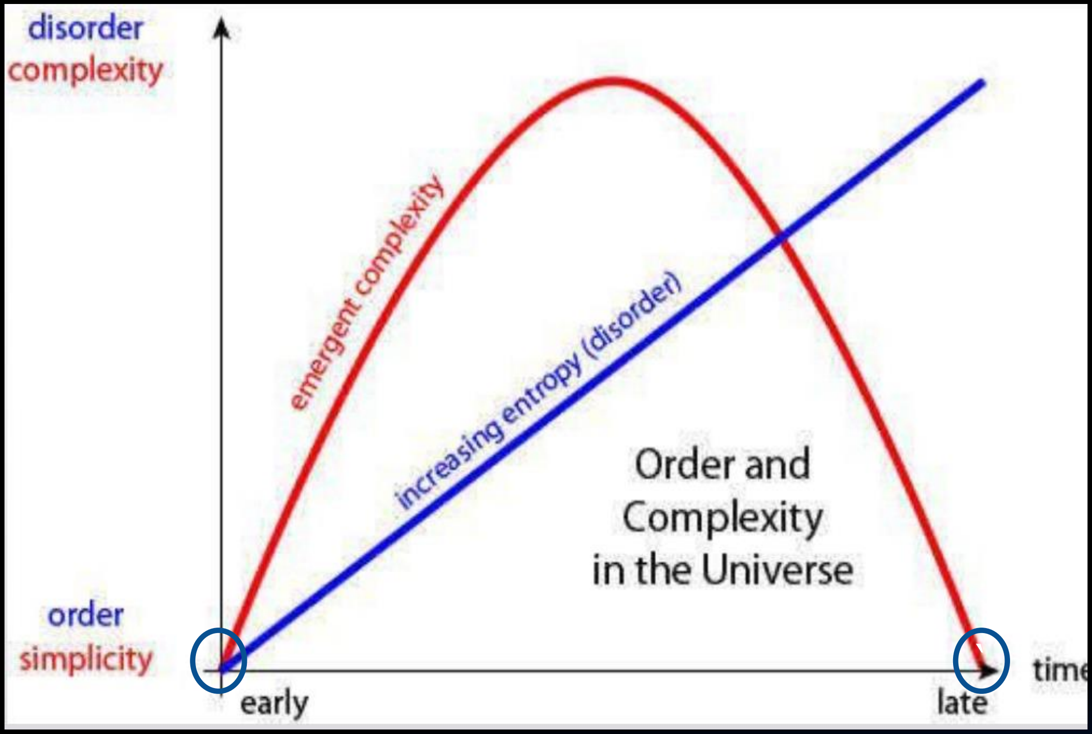
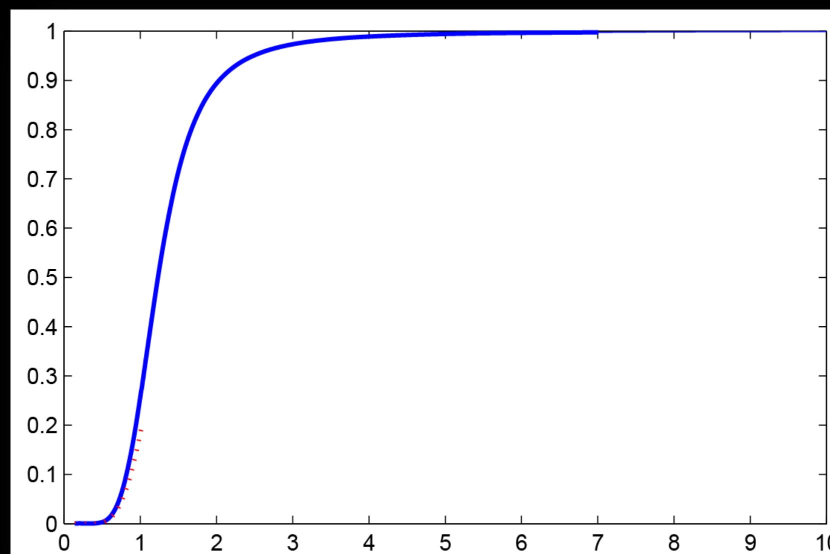
Statistical Complexity



Spin Chain at Temperature T

$$H = -J \sum s_j s_{j+1} - B \sum s_j$$

Complexity

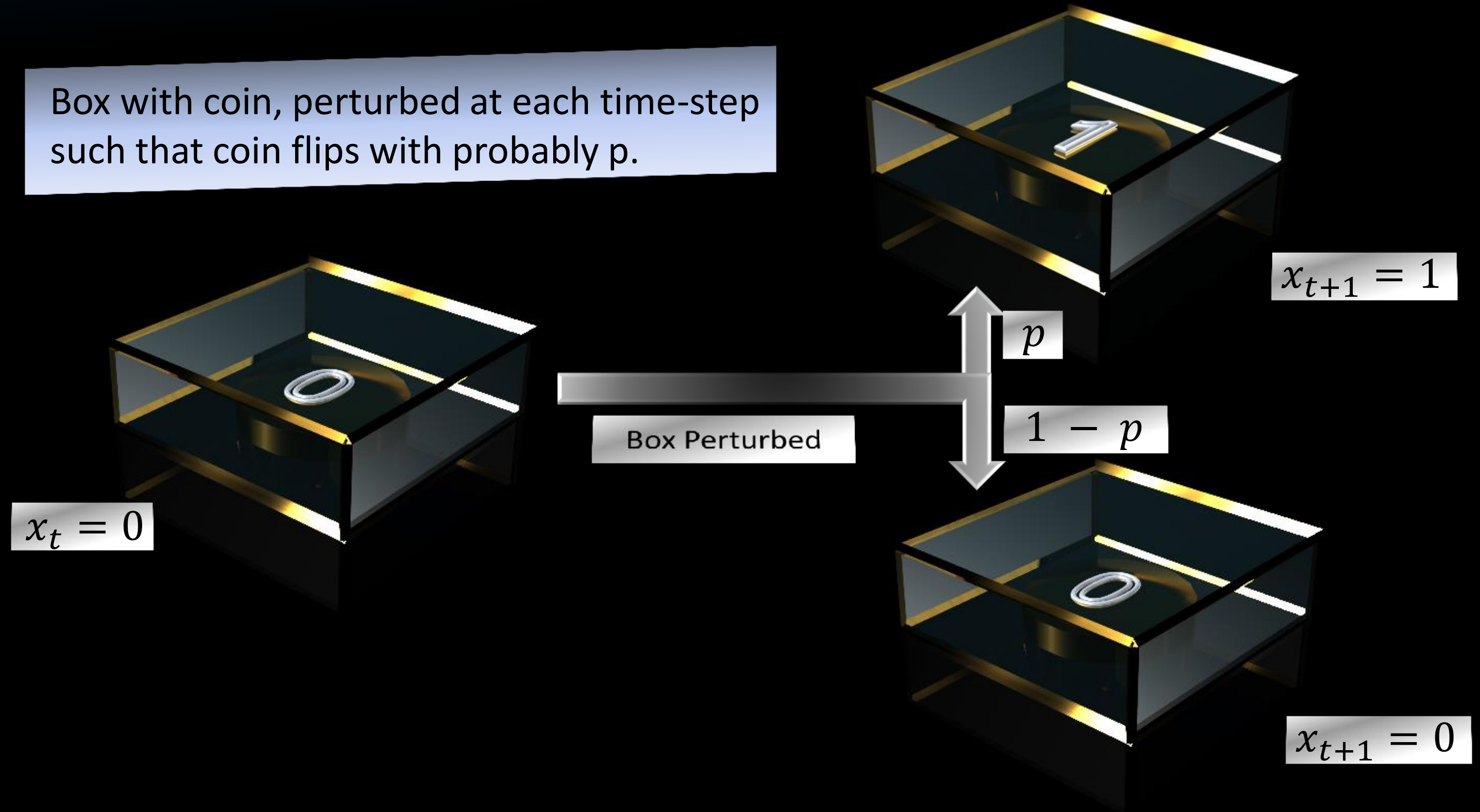


Temperature

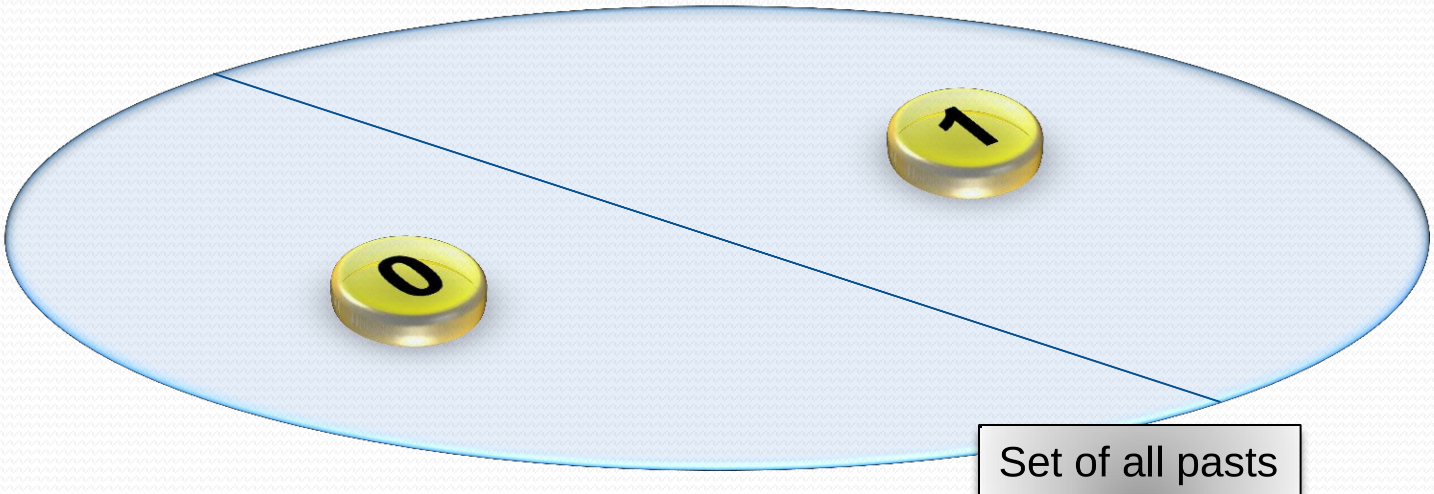
Crutchfield, J., Feldman, D., "Statistical complexity of simple one-dimensional spin systems." Physical Review E 55.2 (1997): R1239.

Example: The Perturbed Coin

Box with coin, perturbed at each time-step such that coin flips with probably p.



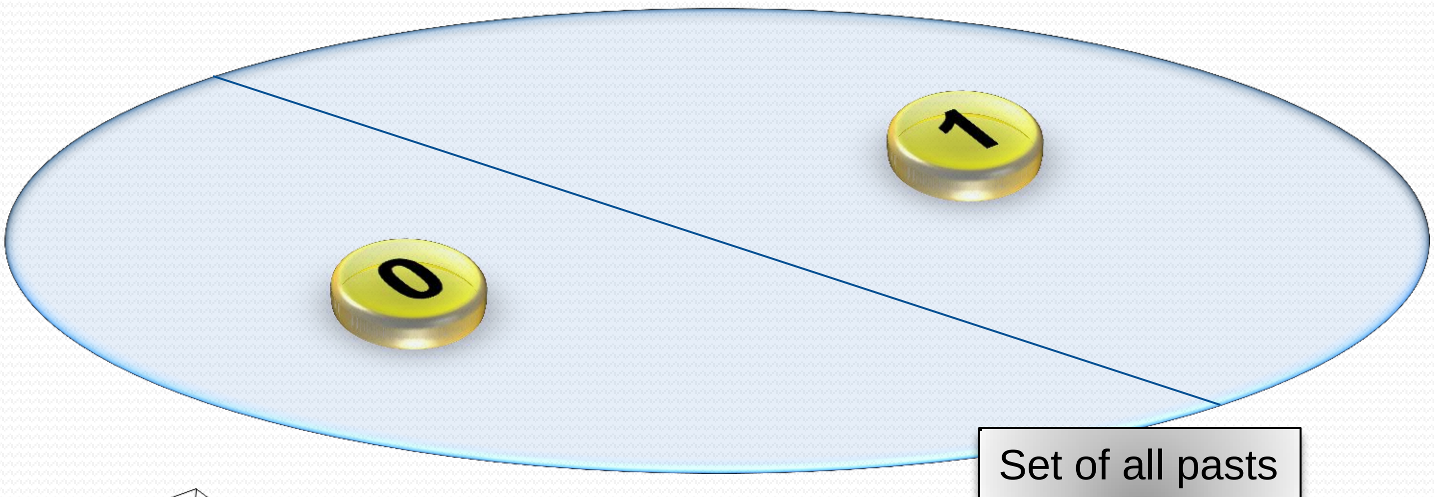
Example: The Perturbed Coin

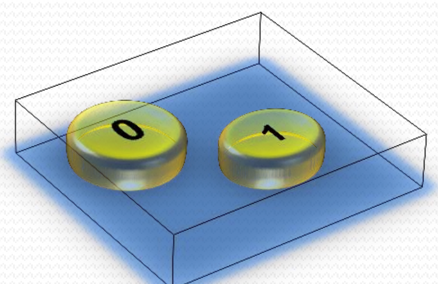


$$P(\vec{X} | 0) \neq P(\vec{X} | 1)$$

We cannot discard information about the state of the coin.

Example: The Perturbed Coin



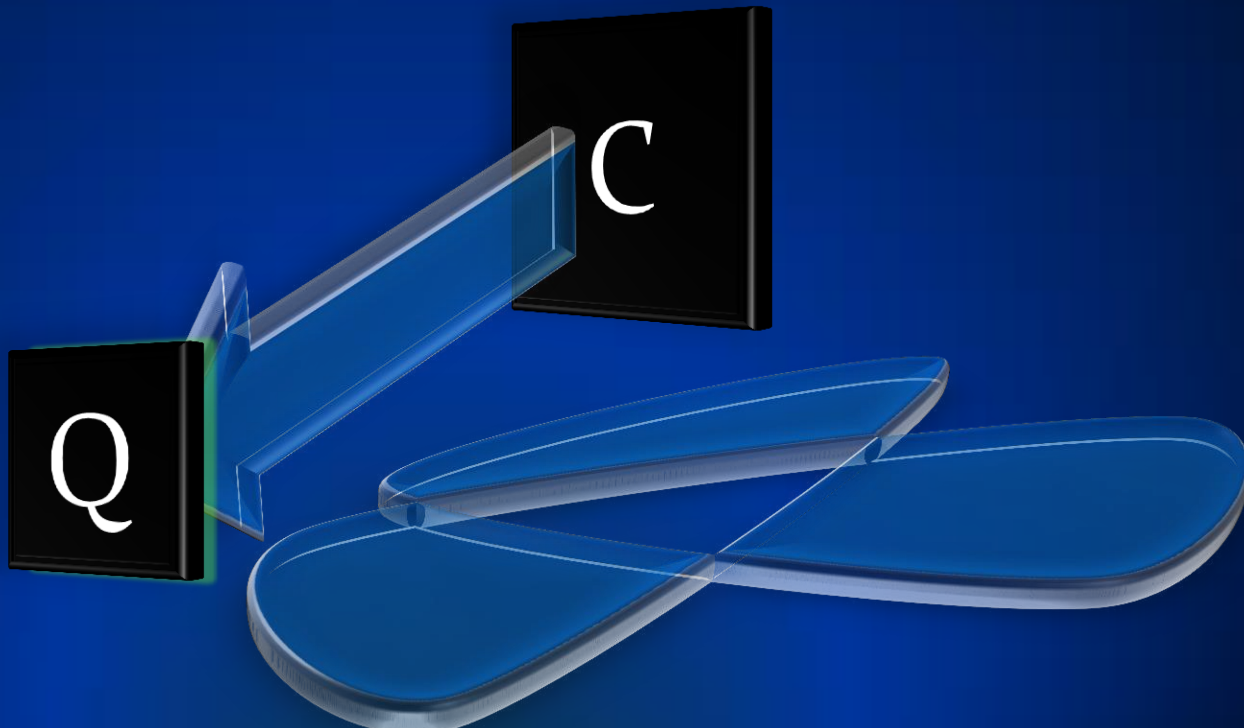


Optimal classical system for generating $\mathbf{P}(\overleftarrow{X}, \overrightarrow{X})$

has internal entropy 1, for any p ≠ 0.5

But as $p \to 0.5$, the process tends towards a completely random sequence!

Can Quantum Logic Provide A Better Description?

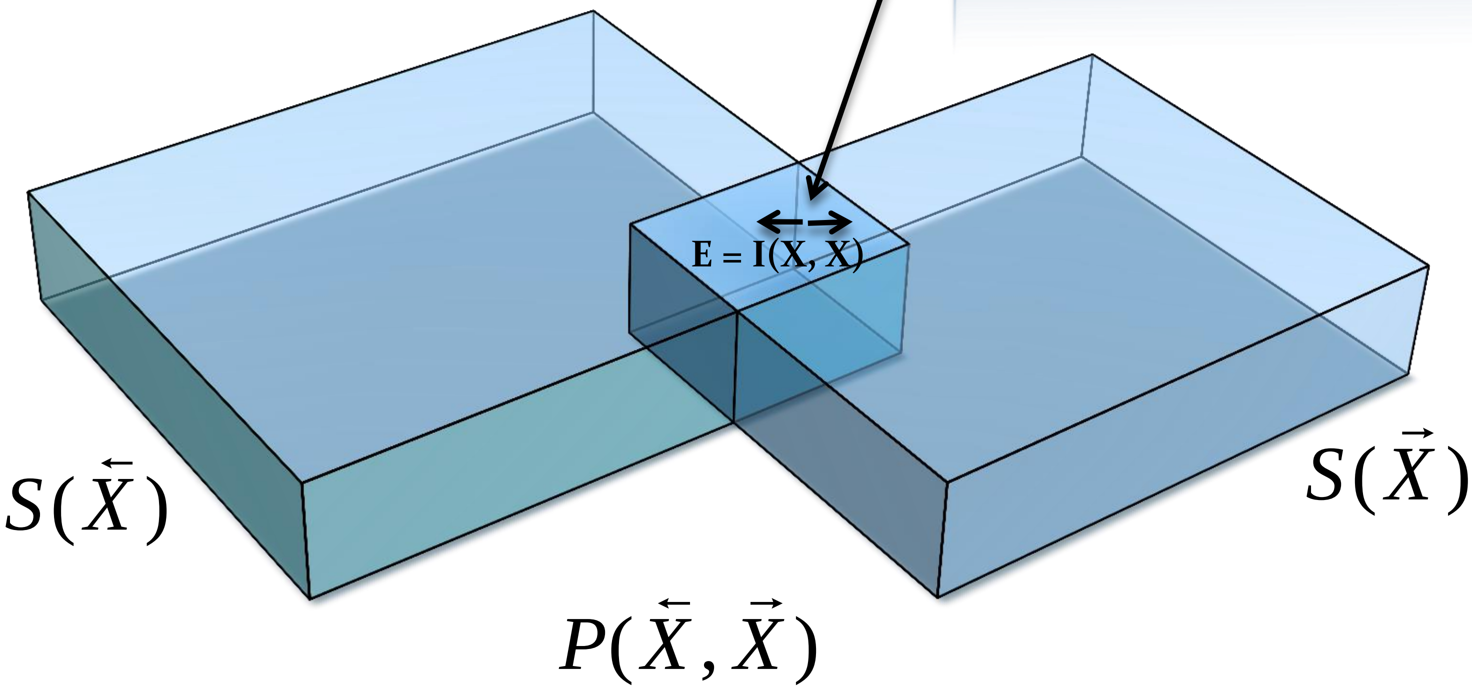


Can Epsilon Machines be improved?

Knowledge of the past contains

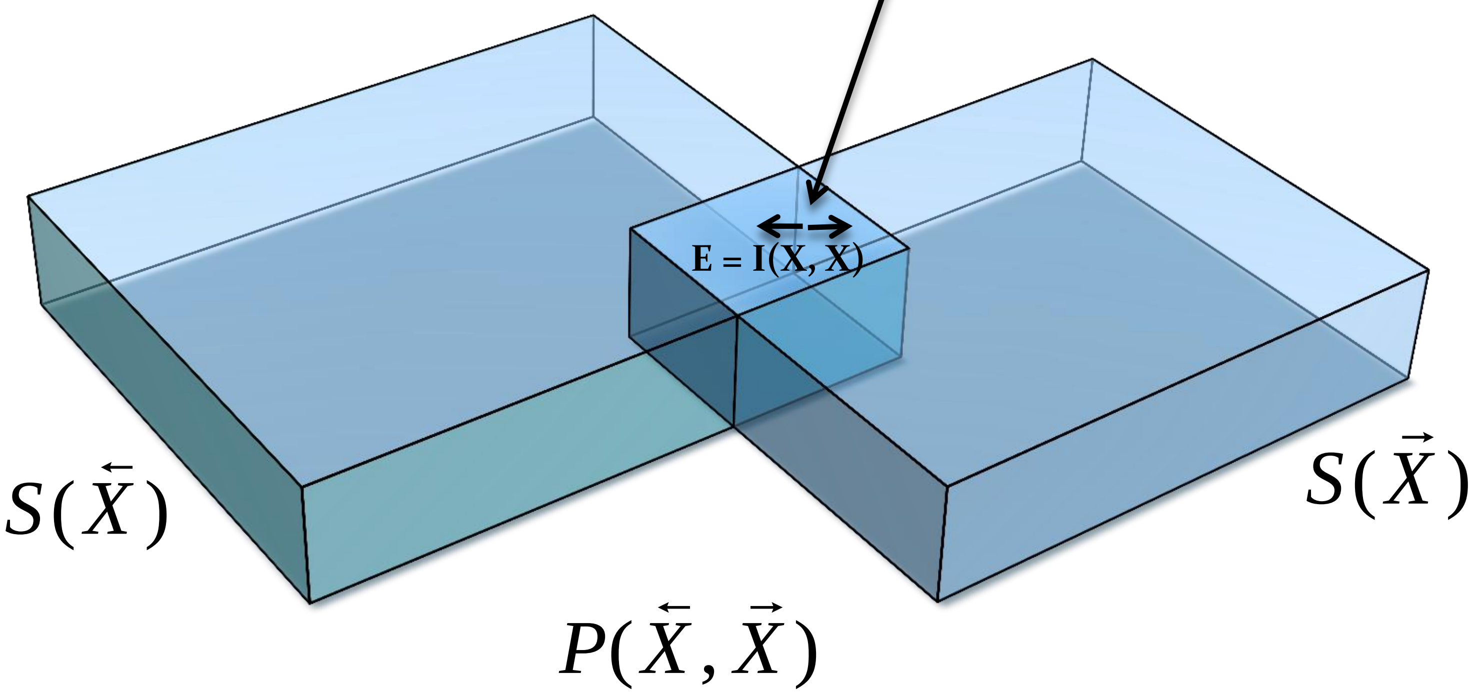
$$S(\vec{X}) - S(\vec{X} \mid \overleftarrow{X}) = I(\vec{X} \mid \overleftarrow{X})$$

bits about the future.



Can Epsilon Machines be improved?

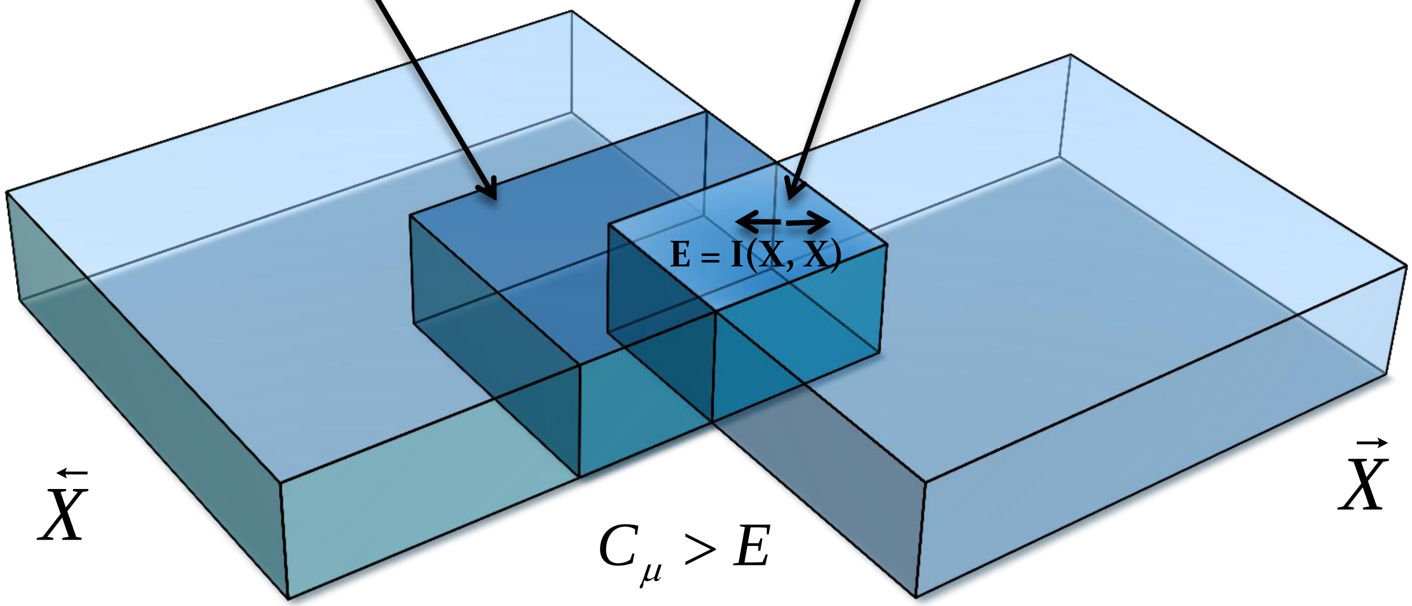
Lower bound on how many bits we must track to fully model a process



Can Epsilon Machines be improved?

An epsilon machines must store C_μ bits to know to retain all information About the future..

Lower bound on how many bits we must track to fully model a process

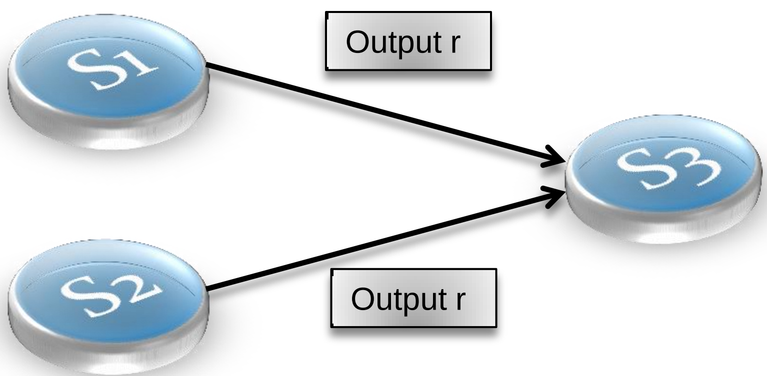


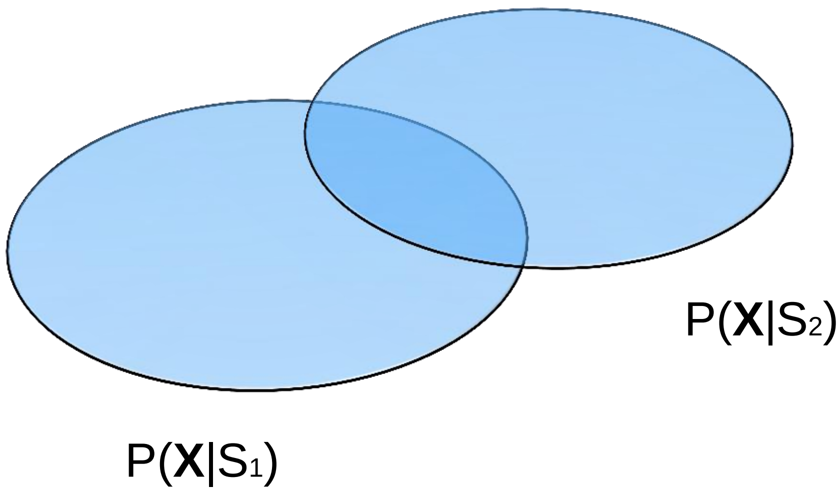
JP **Crutchfield**, CJ Ellison, “Time’s Barbed Arrow.” PRL 2009 - APS

The Optimal Classical Model of a Process still Invokes extraneous causes!

The Source of Extraneous Cause

Suppose two differing causal states have finite probability to transition to an coinciding causal state after coinciding output.





The future of the two causal states are not entirely distinct.

A classical epsilon must store a property A that distinguishes S_1 and S_2. But observation of the entire future does not guarantee the ability to retrieve A.

Some of the storage used to keep track of A is wasted.

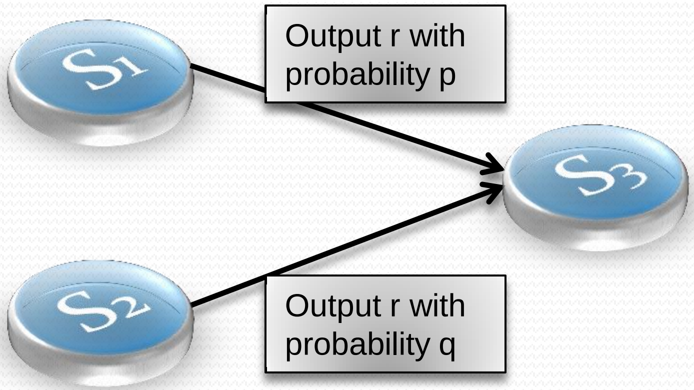
Quantizing Occam's Razor

A quantum epsilon machines does not distinguish the causal states completely.

Map each causal state Sj to a corresponding quantum state

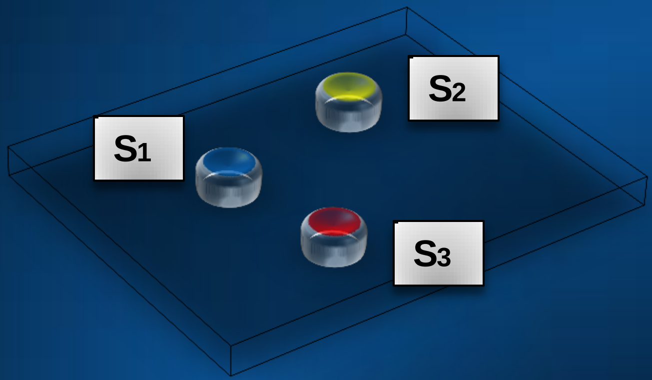
$$\left|S_j\right\rangle = \sum_r \sum_k \sqrt{T^r{}_{j,k}}\left|r\right\rangle\left|k\right\rangle$$

Probability a Stochastic process in Causal state Sj will emit output `r' and transition to Sk



$$\left\langle S_1 \middle| S_2 \right\rangle \geq \sqrt{pq} > 0$$

Storage of quantum causal states require less memory than their classical counterpart



Classical models must allocate enough storage to distinguish every causal state

Quantum systems can go beyond this by compressing the information further… distinguishing the causal states only to the degree that they affect the future.

THEOREM

Provided the best Classical simulator for a stochastic process erases some information, there exists a simpler quantum model


nature COMMUNICATIONS

ARTICLE

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DOI: 10.1038/ncomms1761

Quantum mechanics can reduce the complexity of classical models

Mile Gu[1], Karoline Wiesner[2], Elisabeth Rieper[1] & Vlatko Vedral[1,3,4]

M.Gu, K.Wiesner, E.Rieper, V.Vedral, Nature Communications, 3, 762

...0 1 1 1 0 1 0 1 1 1 1

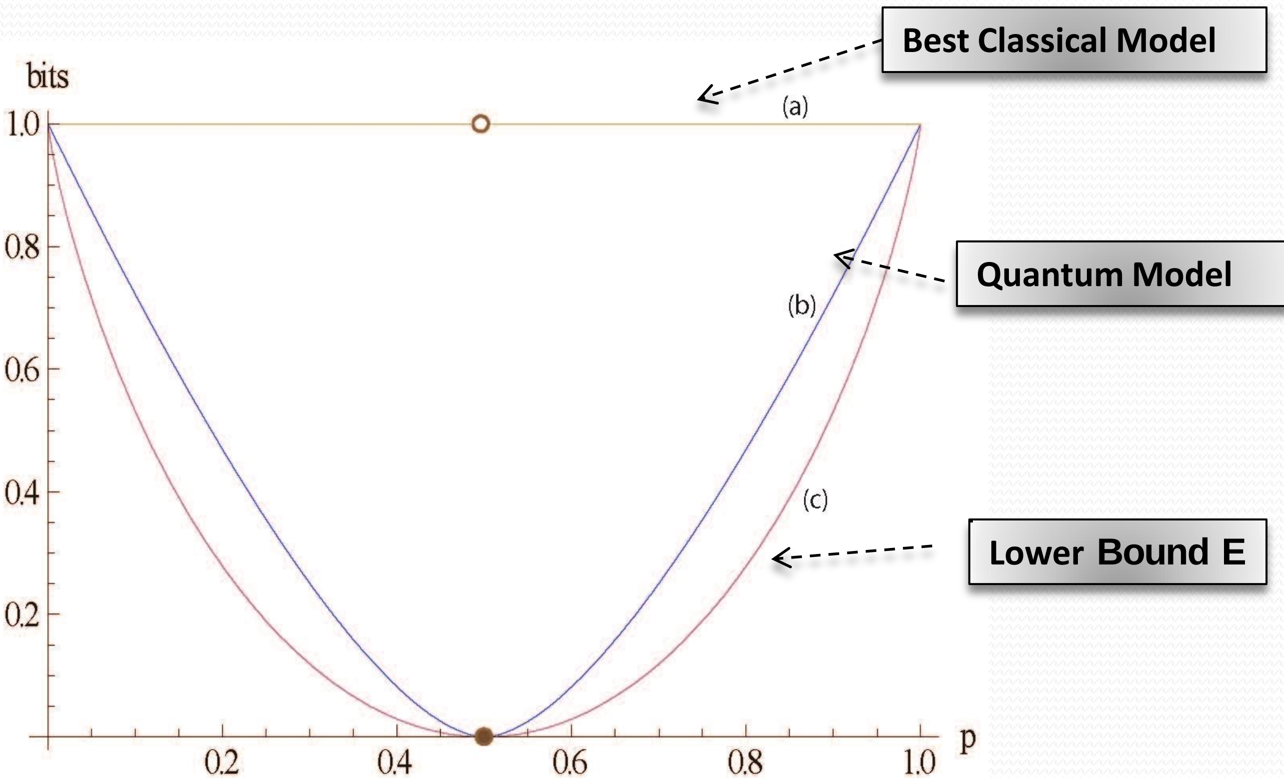
Classical understanding still requires us to perfectly distinguish *Between 0 and 1!*

Simplifying with Quantum

The Quantum Refinement

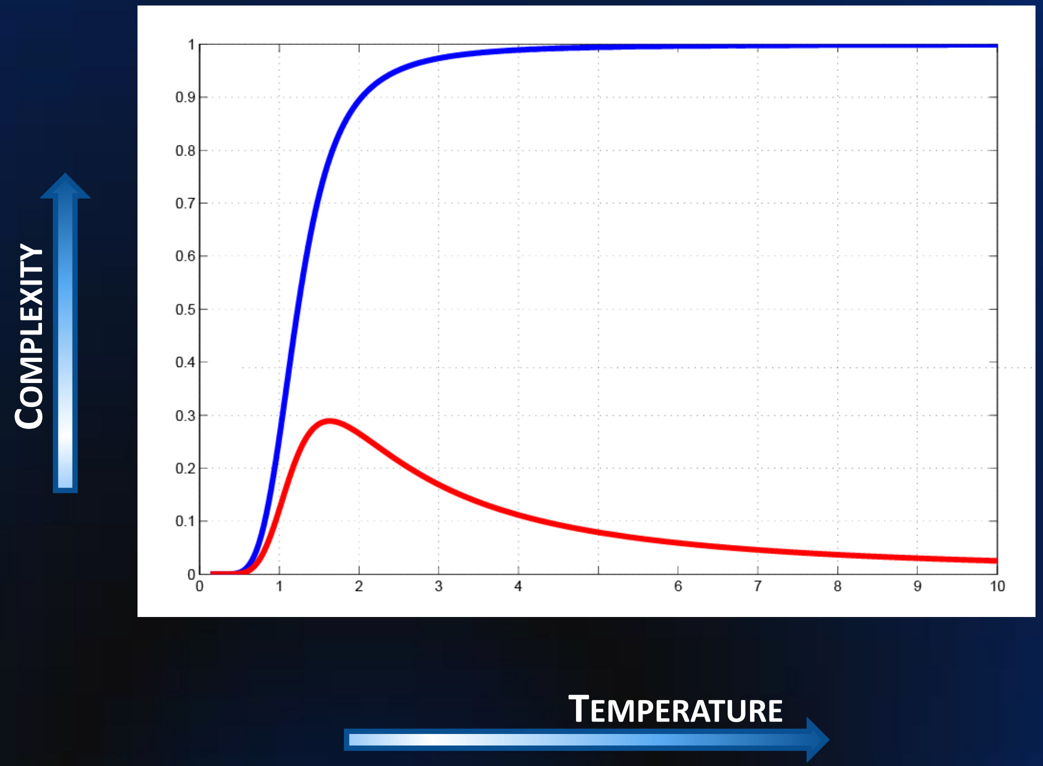
Encode 0 as $|S_0\rangle = \sqrt{1-p}|0\rangle + \sqrt{p}|1\rangle$

Encode 1 as $|S_1\rangle = \sqrt{p}|0\rangle + \sqrt{1-p}|1\rangle.$



Spin Chain at Temperature T

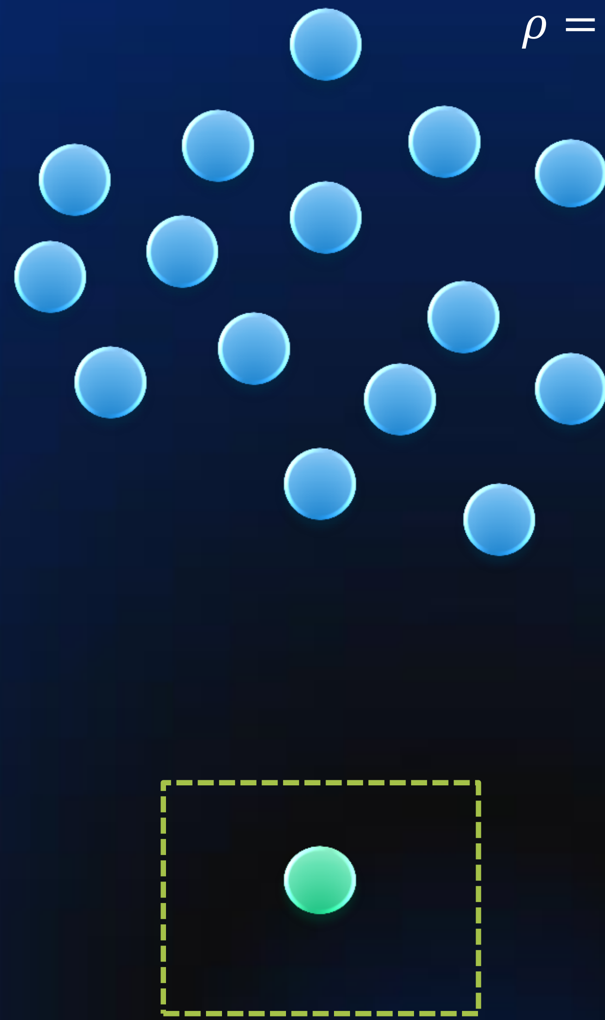
$$H = -J \sum s_j s_{j+1} - B \sum s_j$$

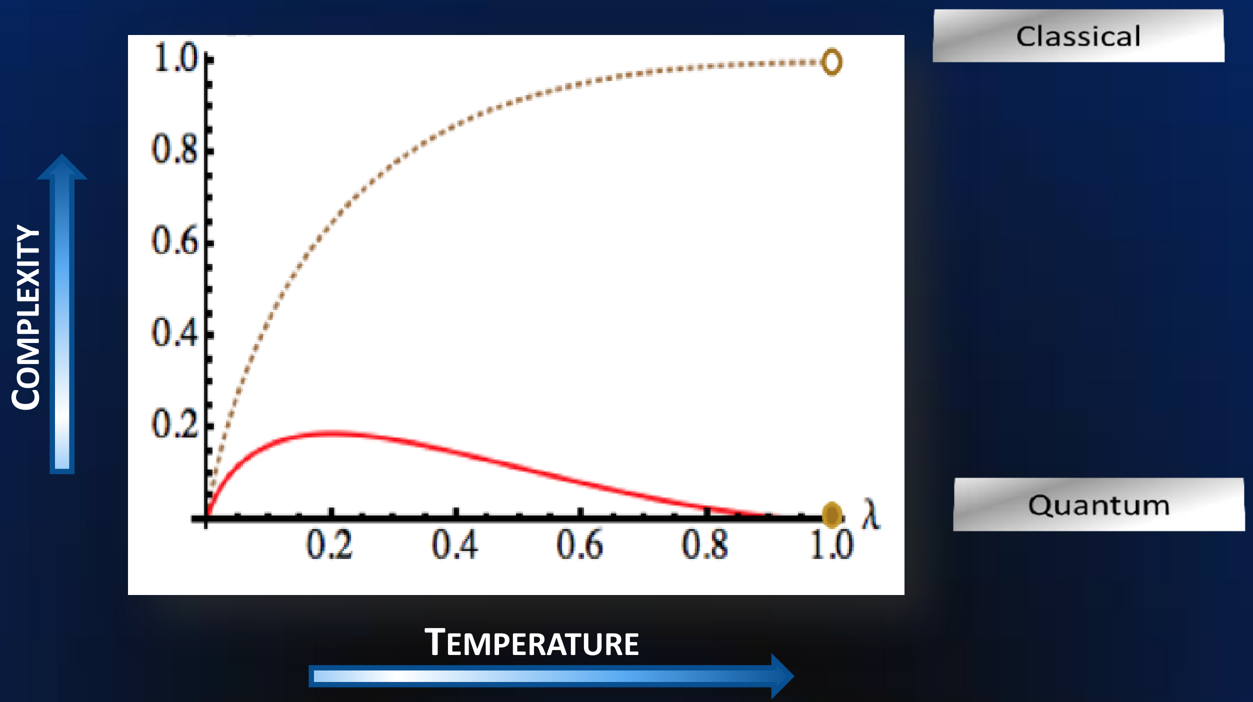


Sun Wei Yeap

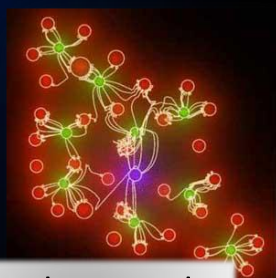
Jayne Thompson

$$\rho = \lambda|0\rangle\langle 0| + (1-\lambda)|1\rangle\langle 1|$$




Classical
Quantum
Complexity
Temperature
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
λ

Outlook



Neural Networks

IEEE Trans. Neural Networks, 10, 2, 284-302

Dripping Faucets

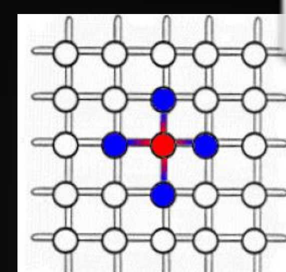
Physica A: 257, 1-4, 385-389

What Happens when we allow Quantum logic?

Pseudo-random Number generators.

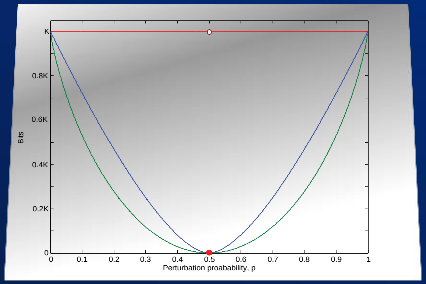
Physica A: 356, 1, 133-138

Ising Models



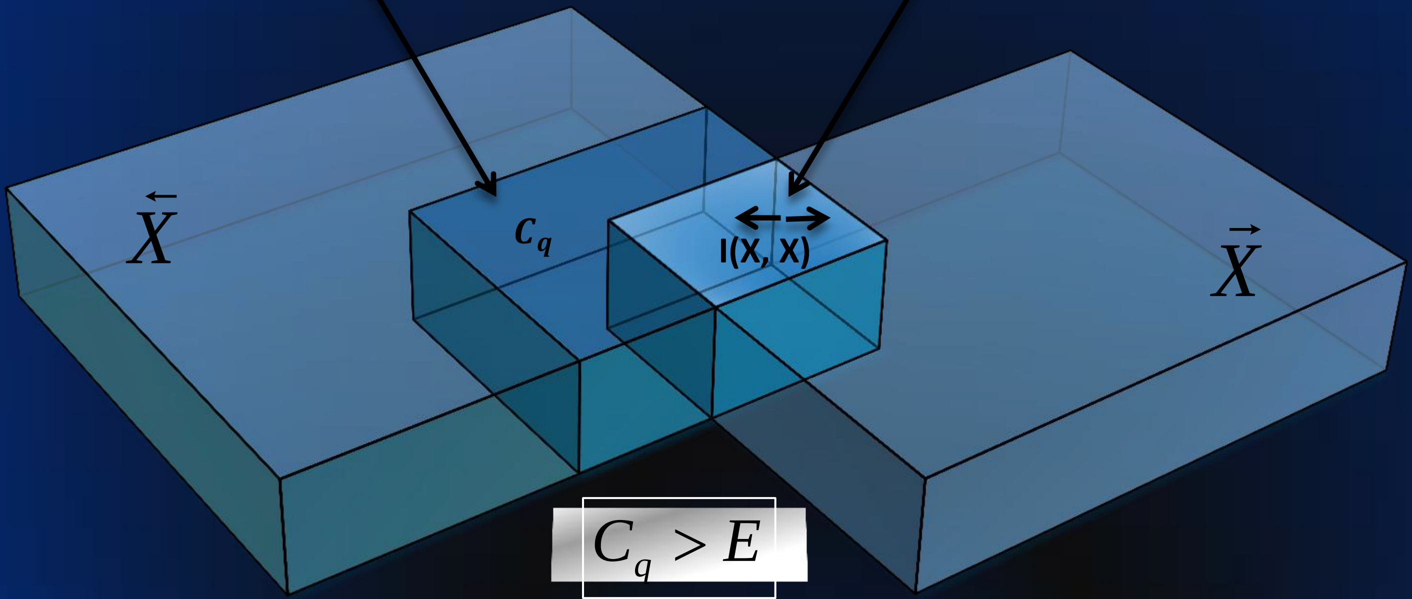
PRA 238, 4-5, 244-252

Mysteries....



Even quantum simulators seem to store unnecessary information

The amount of useful information the past contains about the future.



$$\boxed{C_q > E}$$

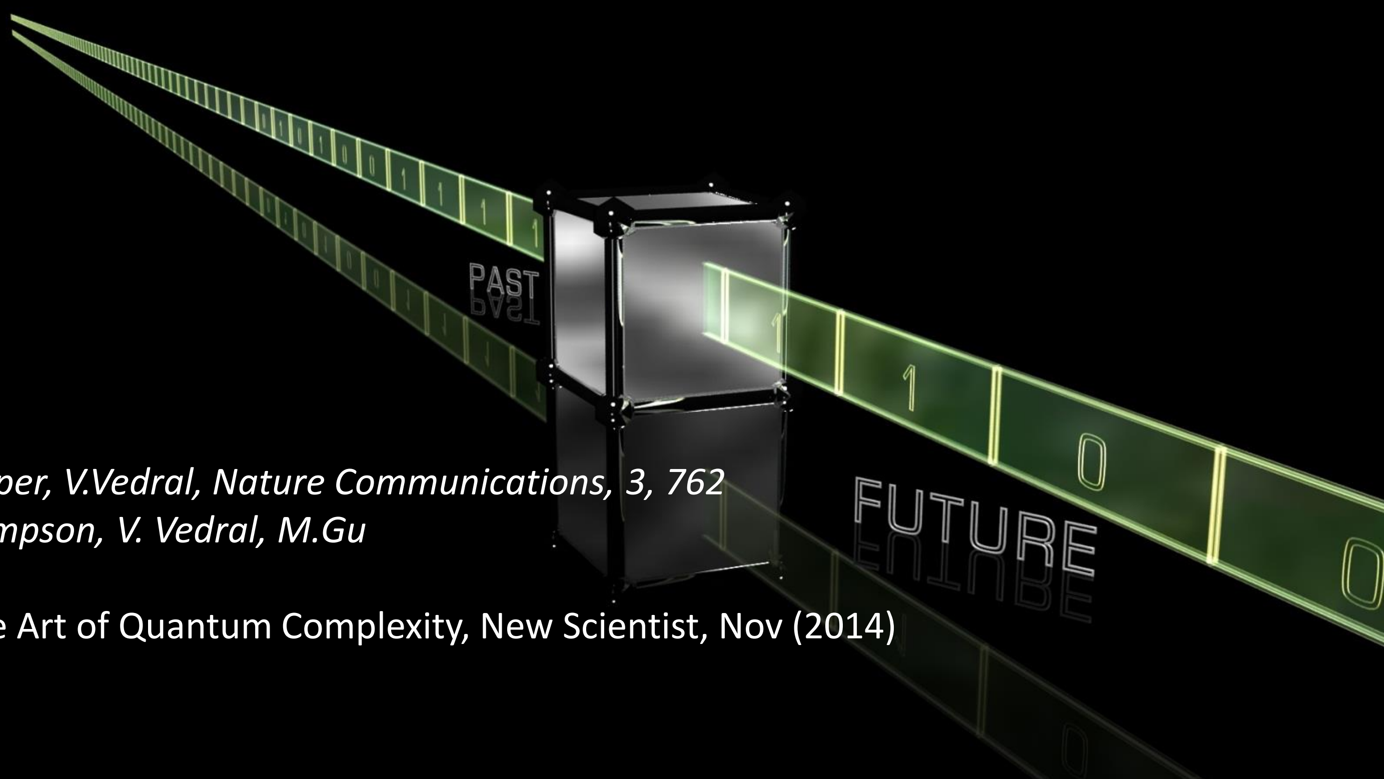
?

Is this an unavoidable source of irreversibility when constructing quantum simulations?

?

Can we formulate even more powerful theories of information that surpass quantum mechanics?

References



Publications

- *M.Gu, K.Wiesner, E.Rieper, V.Vedral, Nature Communications, 3, 762*
- *R. Tan, D. Terno, J. Thompson, V. Vedral, M.Gu*
  EPJ Plus, 129:191
- Gu, Vedral, Zen and the Art of Quantum Complexity, New Scientist, Nov (2014)

Public Article on FQXI:

"If God were to simulate reality, would he prefer it quantum?"
http://fqxi.org/community/forum/topic/1248