Generating solitons on discrete Heisenberg chains and how to manipulate qubits with them

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Motivation

Solitons are non-linear excitations with specific space and time localization; they propagate along one-dimensional media with constant shape and velocity, and are further characterised by stability against noise and perturbations. These features make them particularly suitable for some practical applications, such as the transmission of signals between distant parties, provided a physical support and a method to bear them.

Idea

Find a scheme in order to practically achieve the generation of soliton-like excitations on discrete classical Heisenberg chain by applying a time-dependent magnetic field to one of the chain ends.

The Model

Isotropic Heisenberg classical Hamiltonian plus a boundary term

$$H_{\text{chain}} = -J\sum_n \mathbf{S}_n \cdot \mathbf{S}_{n+1} - \gamma B \sum_n S_n^z - \gamma \mathbf{S}_1 \cdot \mathcal{B}(t)$$

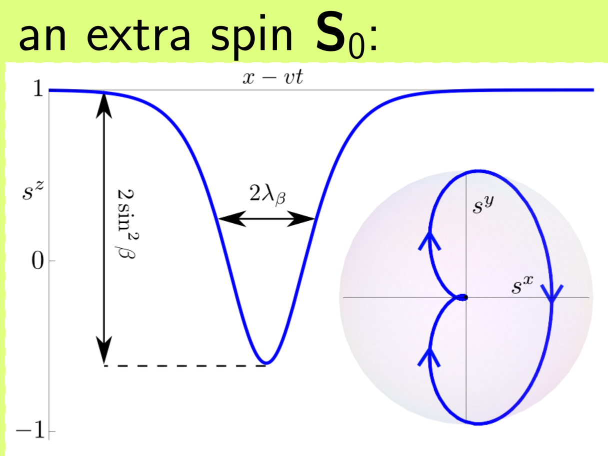
J is the chain coupling, γ the gyromagnetic ratio of the spins, B a uniform magnetic field along z-axis and n runs from 1 to the chain length N.

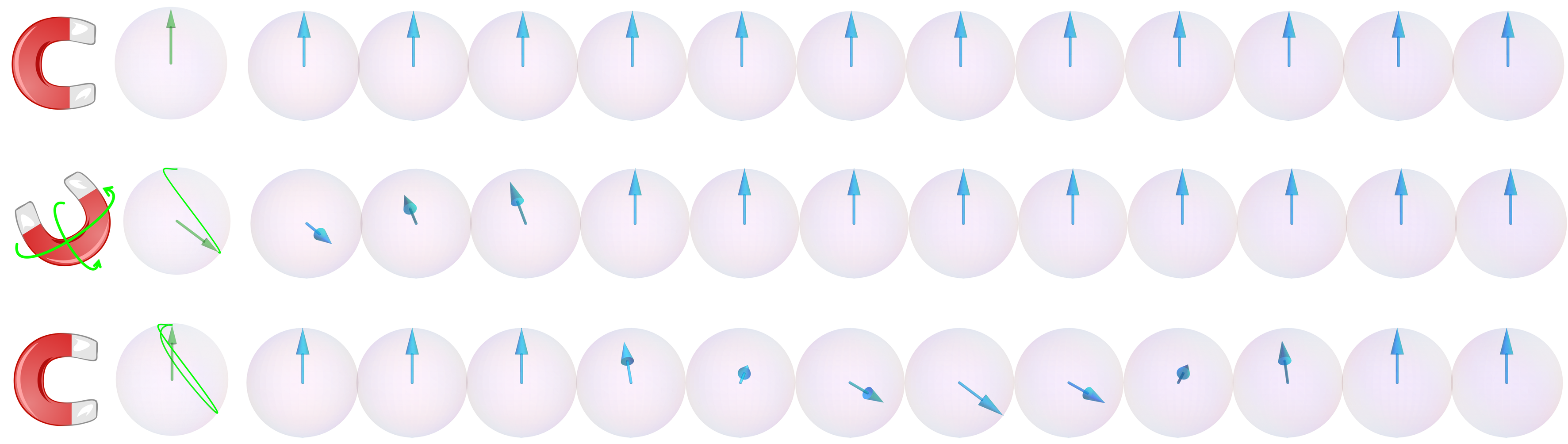
Boundary conditions

Magnetic field at the edge of the chain mimics an extra spin $\mathbf{S}_0$:

$$\gamma\mathcal{B}(t) = J\mathbf{S}_0(t,\beta)$$

Time dependence is given by the analytical soliton solution of continuous chain [1], which depends on a new parameter β. The other edge of the chain is open.





Chain Dynamics

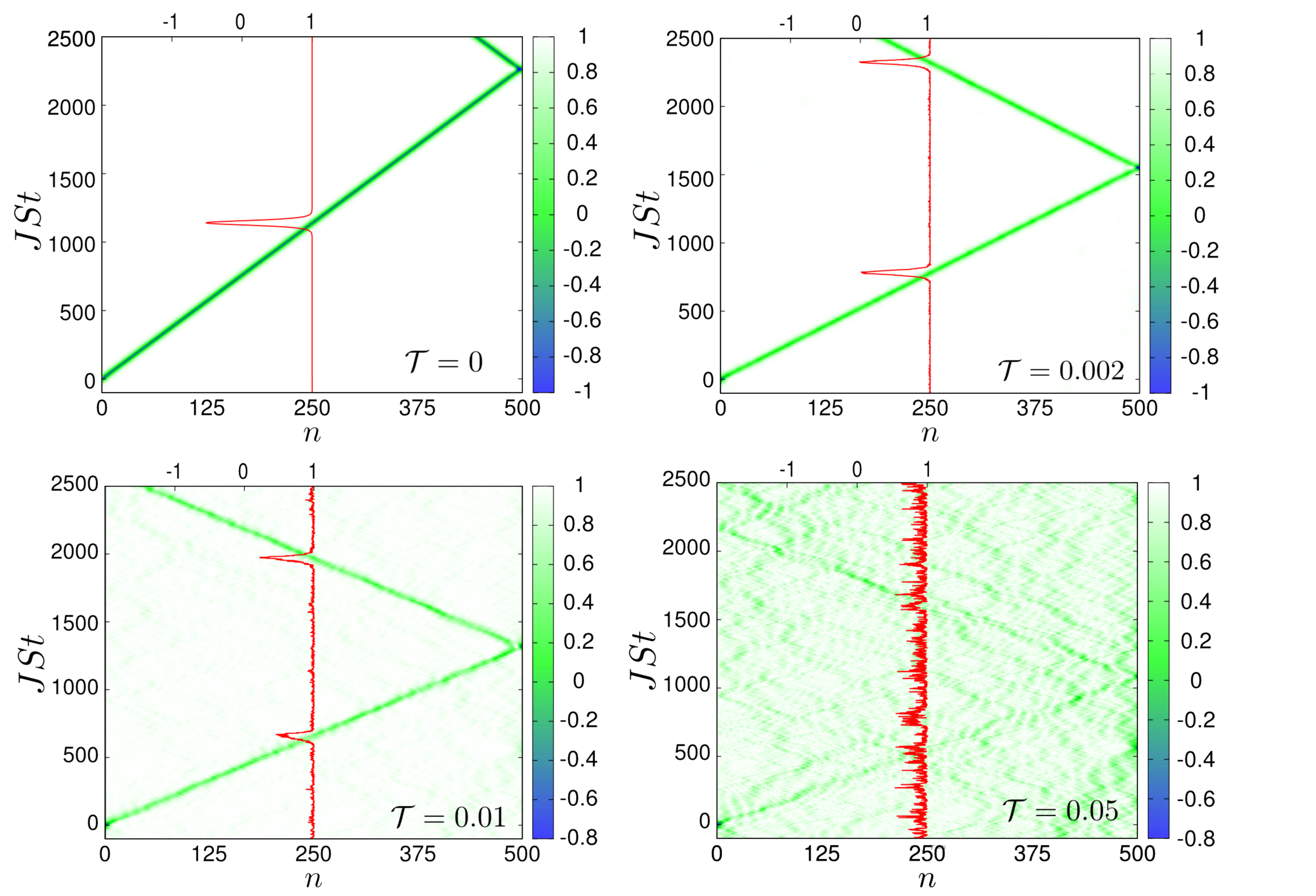
Chain dynamics has been numerically simulated by integrating EOM from the model Hamiltonian with a second order symplectic algorithm [2, 3, 4].

Initial Conditions

- Zero temperature $\mathcal{T} \equiv \dfrac{k_B T}{J\mathcal{S}^2} = 0$ case:
  Ferromagnetic minimum energy configuration, $\{\mathbf{S}_n = \mathcal{S}\hat{\mathbf{z}},\ n=1,\dots,N\}$.
- Finite temperature $\mathcal{T} \neq 0$ case: Typical finite temperature configuration built up so that magnetization fluctuations in Fourier space are directly proportional to $\mathcal{T}$.

Main results

- Constant shape and velocity excitations are indeed generated and show strong stability in spite of propagating on a discrete chain, of free edge reflection and of thermal noise.
- Generated excitation looks similar to Heisenberg solitons from [1]: Fitting data with analytical shape leads to estimated parameters which agrees with those observed. Also relations among parameters β, reduced field $\gamma B/(J\mathcal{S})$ and other quantities (i.e. velocity and energy) almost holds in this case.
- Discreteness effects (i.e. differences between generated excitation and continuous solitons) are less evident for broad solitons.



Current development

- More realistic boundary condition: magnetic field with a finite penetration length (i.e. interacting with more than the first spin).
- Characterization of temperature effects by simulating many noise realizations.

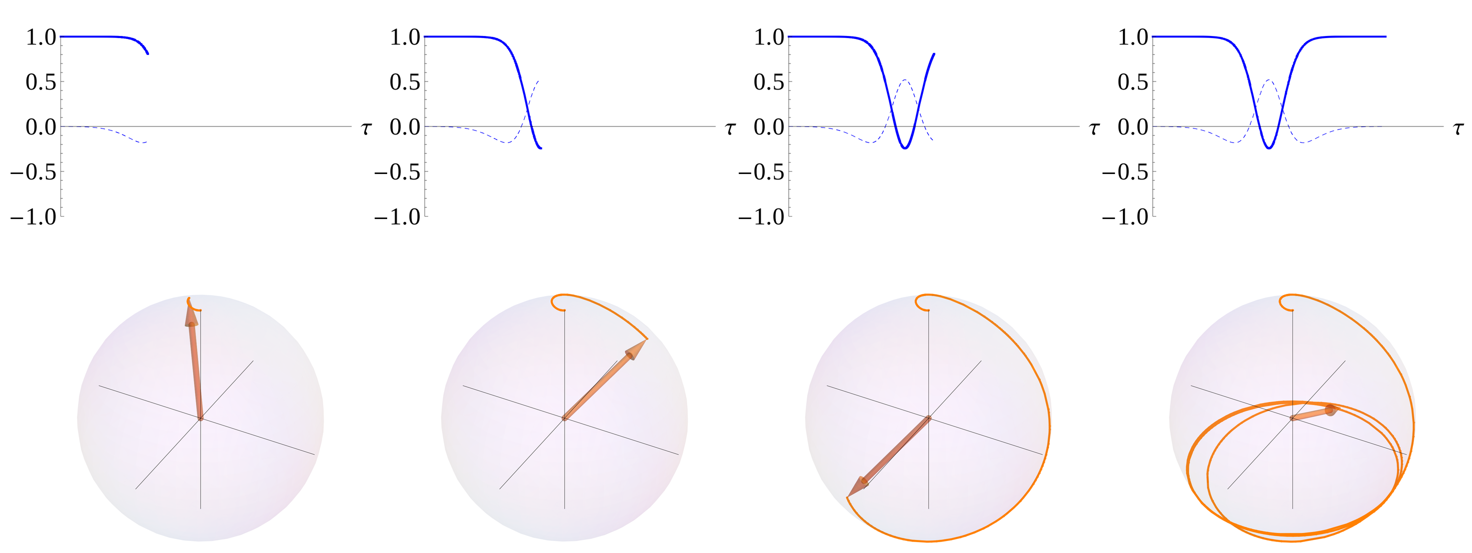
Manipulating a qubit's state

Adding following terms to the Hamiltonian we could model a qubit interacting with a finite number of elements of the spin chain.

$$\hat{H} = H_{\text{chain}} - \gamma_\sigma B\frac{\hbar\hat{\sigma}^z}{2} - g\frac{\hbar\hat{\boldsymbol{\sigma}}}{2}\cdot\sum_n p_n \mathbf{S}_n$$

$\hat{\boldsymbol{\sigma}}$ is Pauli matrices vector, γ_σ is qubit's gyromagnetic ratio, g is the qubit-chain interaction intensity while p_n gives the decreasing (chosen to be gaussian, and such that $\sum_n p_n = 1$) of the interaction with the distance from qubit.

Under no back-action assumption, the qubit feels the propagating soliton as an effective magnetic field. As the soliton runs by, qubit's state is modified.



Relevant parameters

Soliton parameter (β)

Reduced uniform magnetic field $\left(\dfrac{\gamma B}{J\mathcal{S}}\right)$

Interaction constants ratio $\left(\dfrac{g}{J}\right)$

Ratio of the gyromagnetic ratios $\left(\dfrac{\gamma_\sigma}{\gamma}\right)$

Range of qubit's interaction (α)

Results

After soliton transit, qubit is left in an asymptotic state characterized by a constant value of z-component of the magnetization ($\langle\sigma^z\rangle_{\text{out}}$), which is function of the relevant parameters. Particularly interesting is the case of the spin-flip in which the final state is completely determined. Solutions can be divided in two categories:

- Case $\alpha = 0$ [5, 6]: Reduced field parameter can be absorbed and is not relevant, there is only one spin flip point which is $\gamma_\sigma/\gamma = \gamma B/g\mathcal{S} = 1$ regardless for the value of β.
- Case $\alpha \neq 0$ [6]: All the former parameters are relevant: the $\langle\sigma^z\rangle_{\text{out}}$ manifold in parameter space looks more structured and has yet to be entirely characterized.

References

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