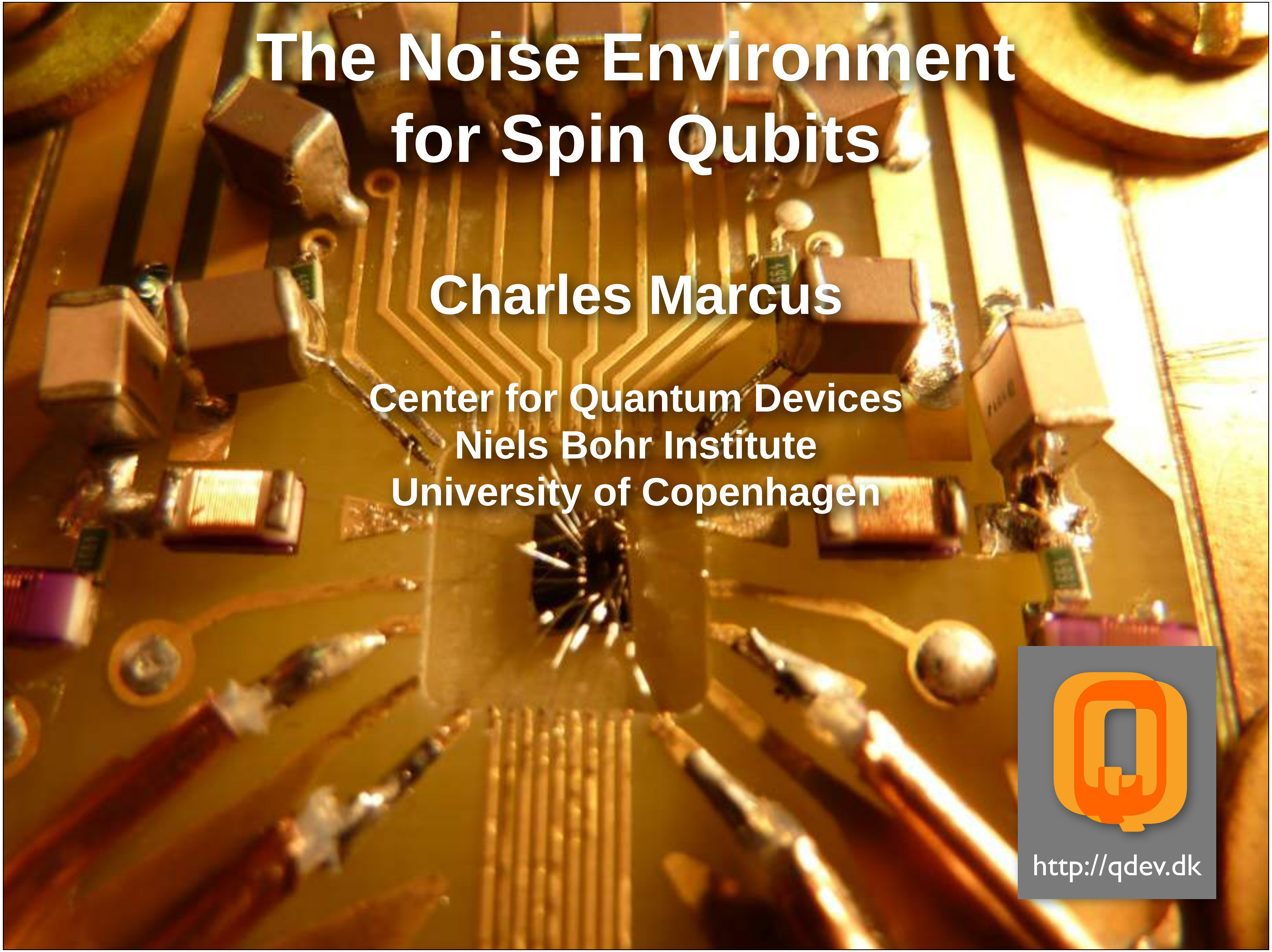


The Noise Environment for Spin Qubits

Charles Marcus

Center for Quantum Devices
Niels Bohr Institute
University of Copenhagen

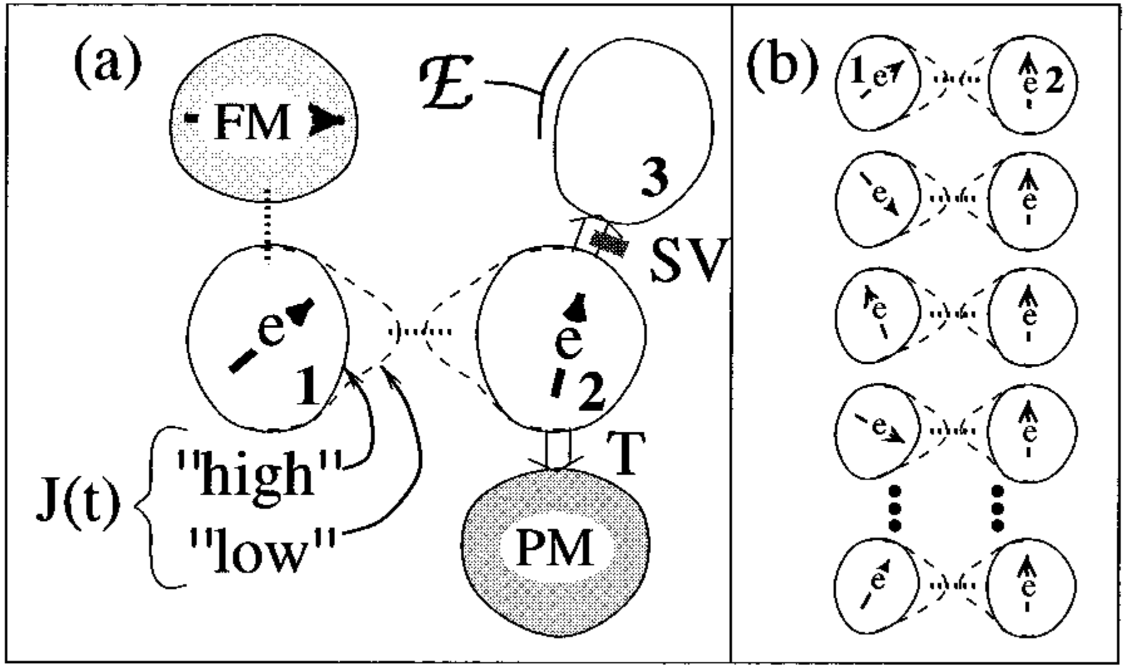
http://qdev.dk

PHYSICAL REVIEW A VOLUME 57, NUMBER 1 JANUARY 1998

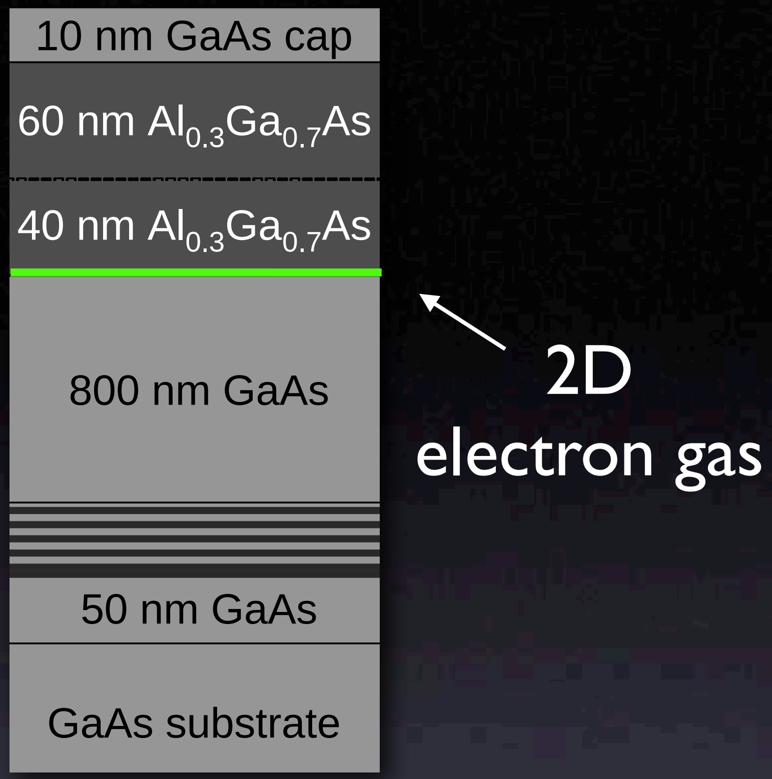
Quantum computation with quantum dots

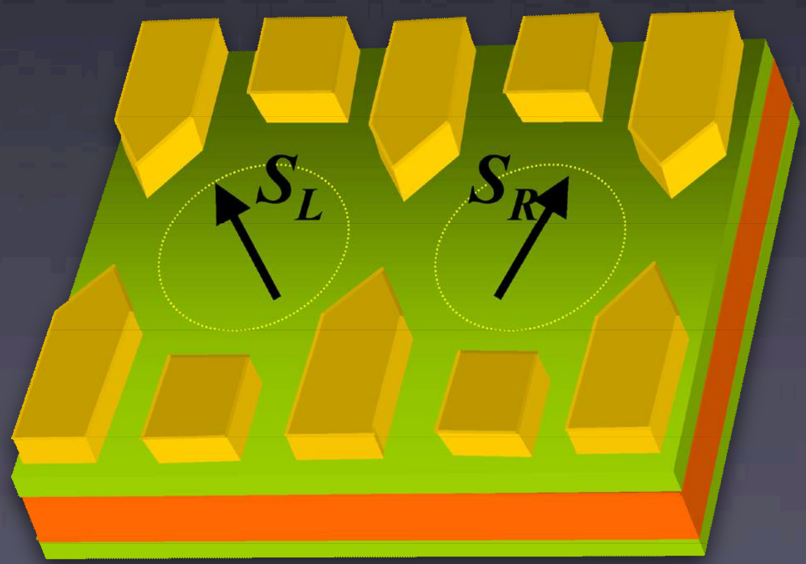
Daniel Loss[1,2,*] and David P. DiVincenzo[1,3,†]
[1]*Institute for Theoretical Physics, University of California, Santa Barbara, Santa Barbara, California 93106-4030*
[2]*Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, 4056 Basel, Switzerland*
[3]*IBM Research Division, T.J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598*
(Received 9 January 1997; revised manuscript received 22 July 1997)

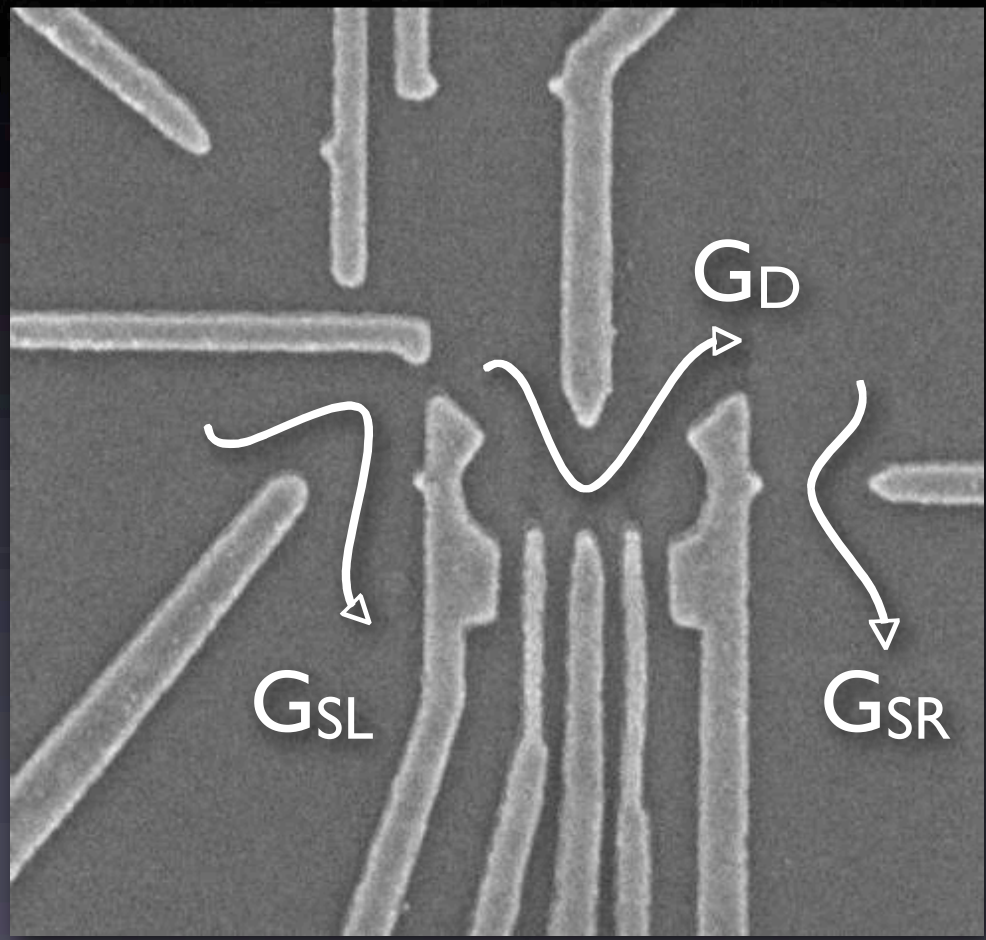
We propose an implementation of a universal set of one- and two-quantum-bit gates for quantum computation using the spin states of coupled single-electron quantum dots. Desired operations are effected by the gating of the tunneling barrier between neighboring dots. Several measures of the gate quality are computed within a recently derived spin master equation incorporating decoherence caused by a prototypical magnetic environment. Dot-array experiments that would provide an initial demonstration of the desired nonequilibrium spin dynamics are proposed. [S1050-2947(98)04501-6]



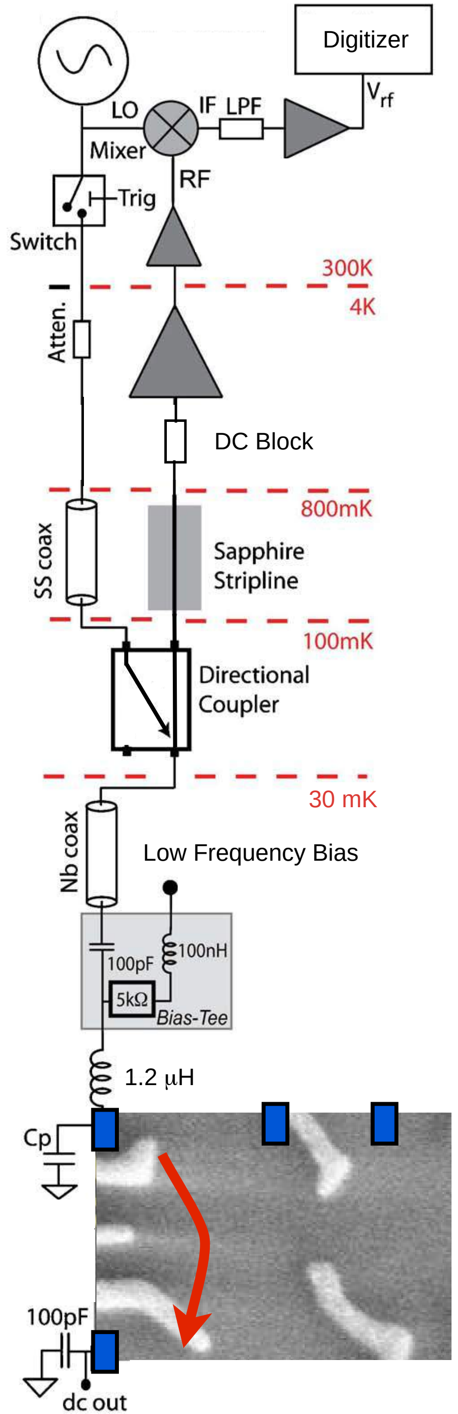
Semiconductor Spin Qubits

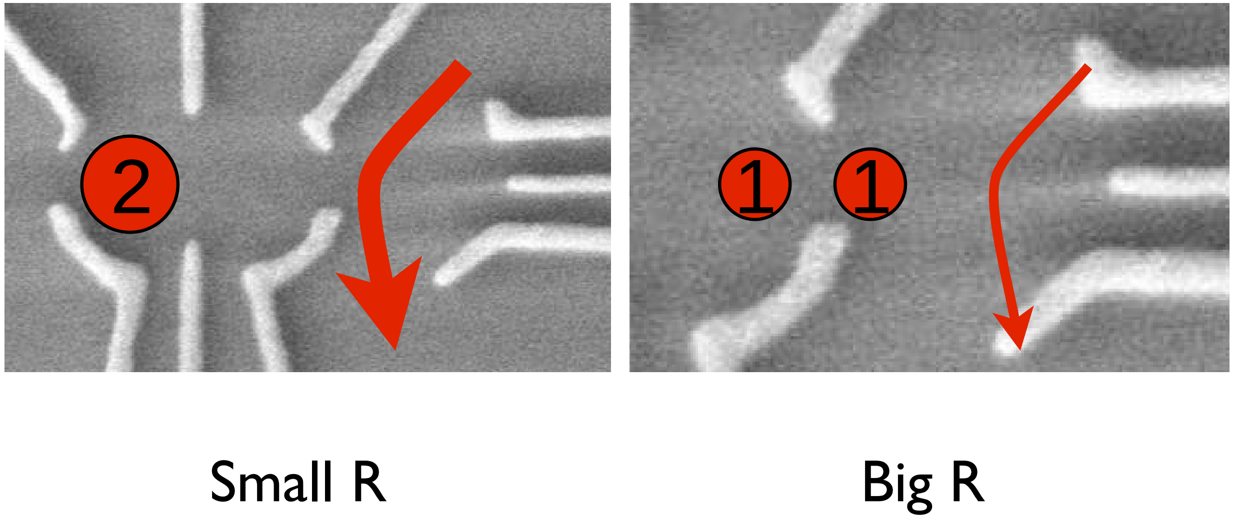


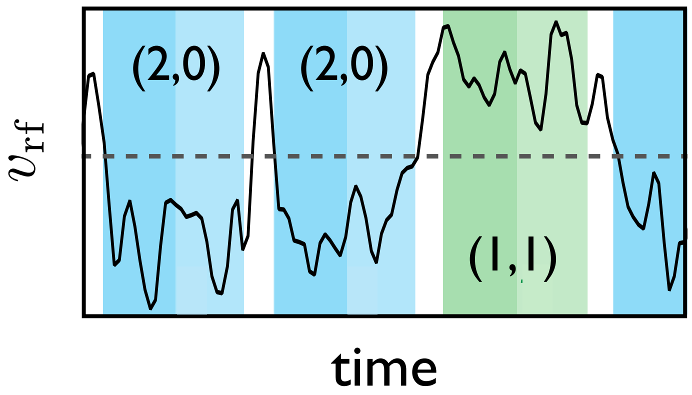


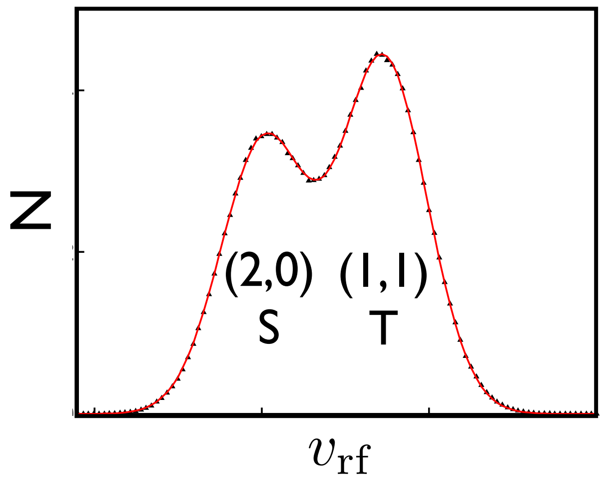


Readout Circuit and Single-shot

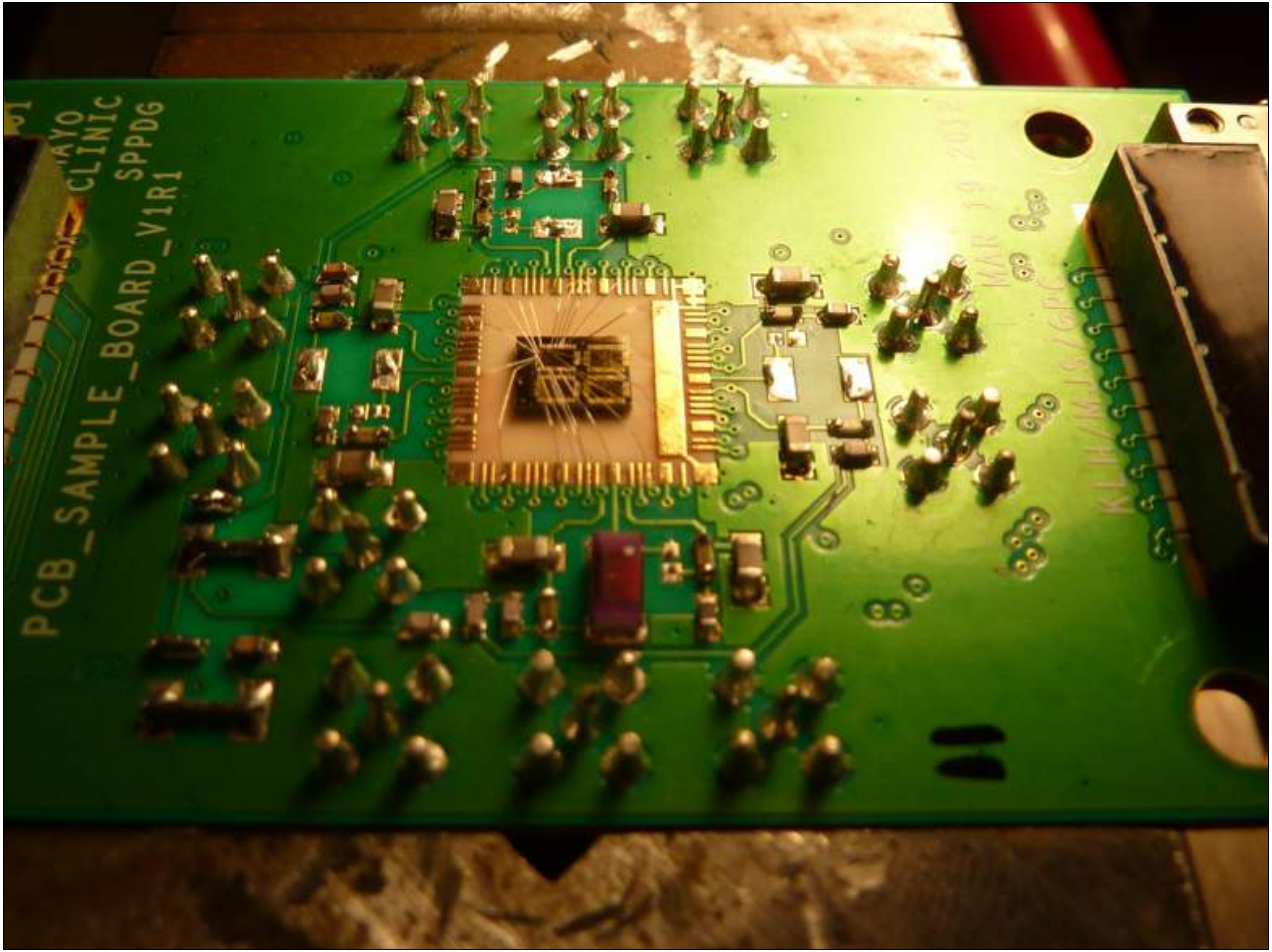








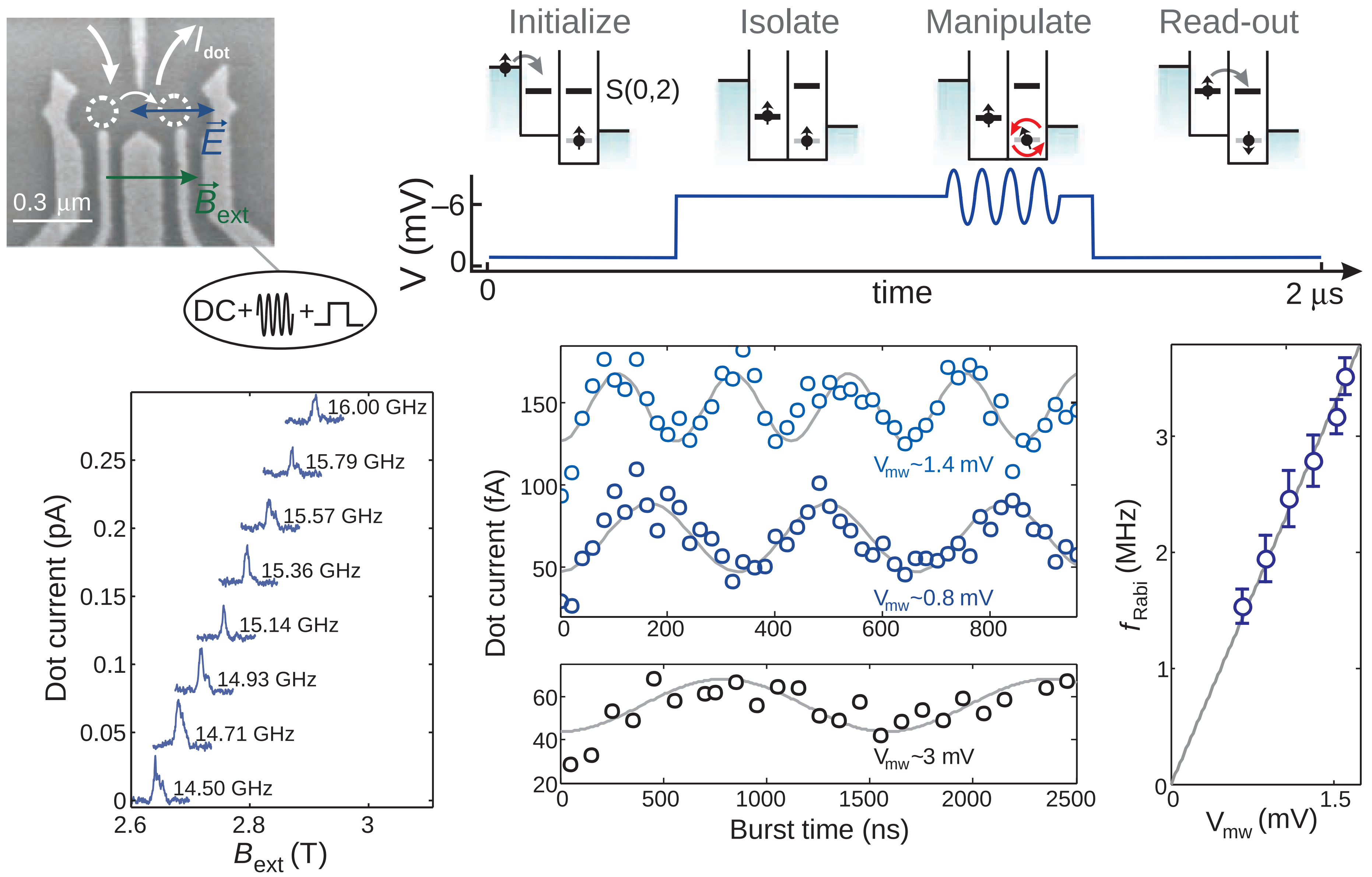
Circuit schematic and voltage trace adapted from Appl. Phys. Lett. 91, 162101 (2007), Phys. Rev. Lett. 103, 160503 (2009)


CLINIC
SPPDG
PCB_SAMPLE_BOARD_V1R1

30 NOVEMBER 2007 VOL 318 SCIENCE

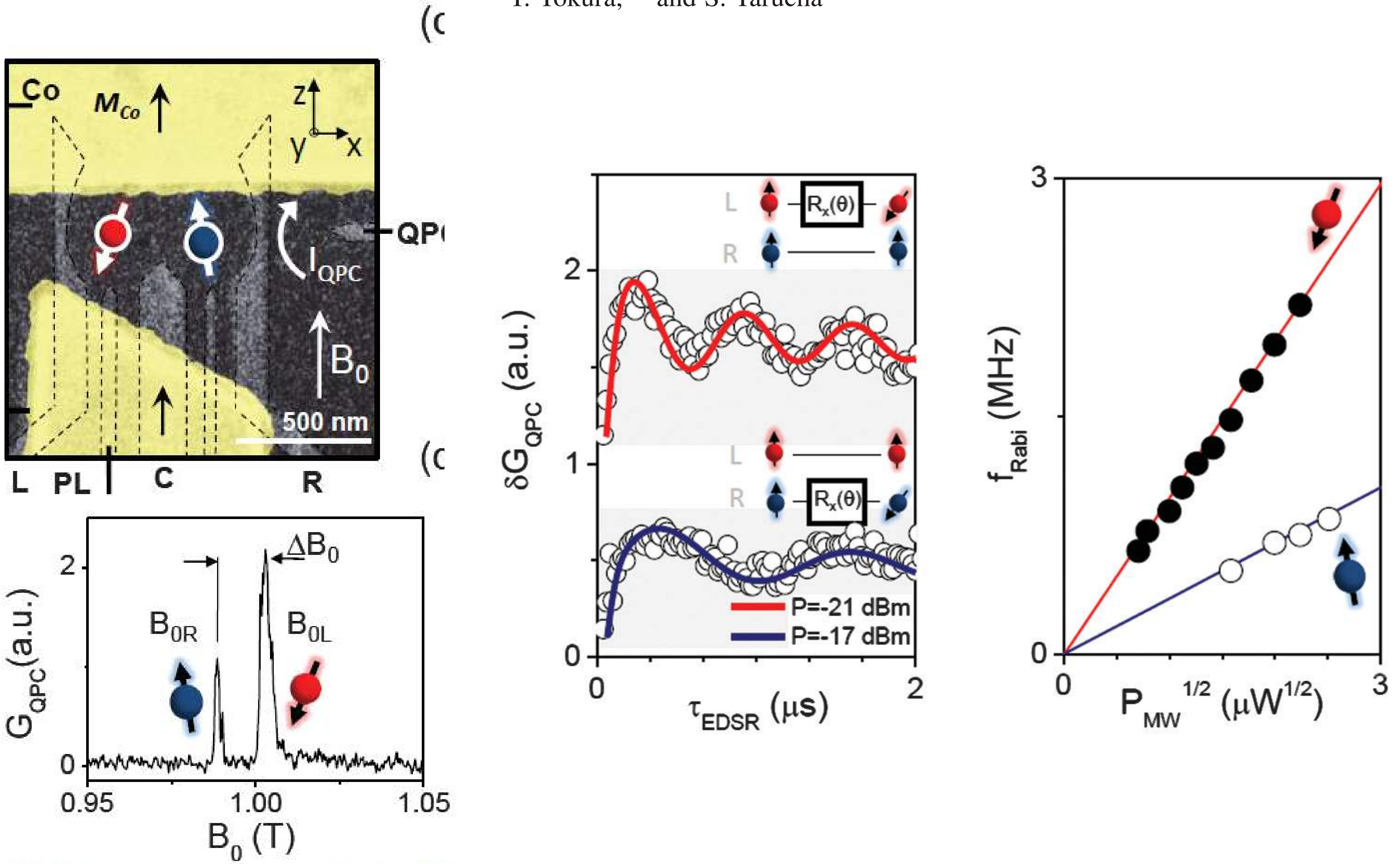
Coherent Control of a Single Electron Spin with Electric Fields

K. C. Nowack,*† F. H. L. Koppens,† Yu. V. Nazarov, L. M. K. Vandersypen*

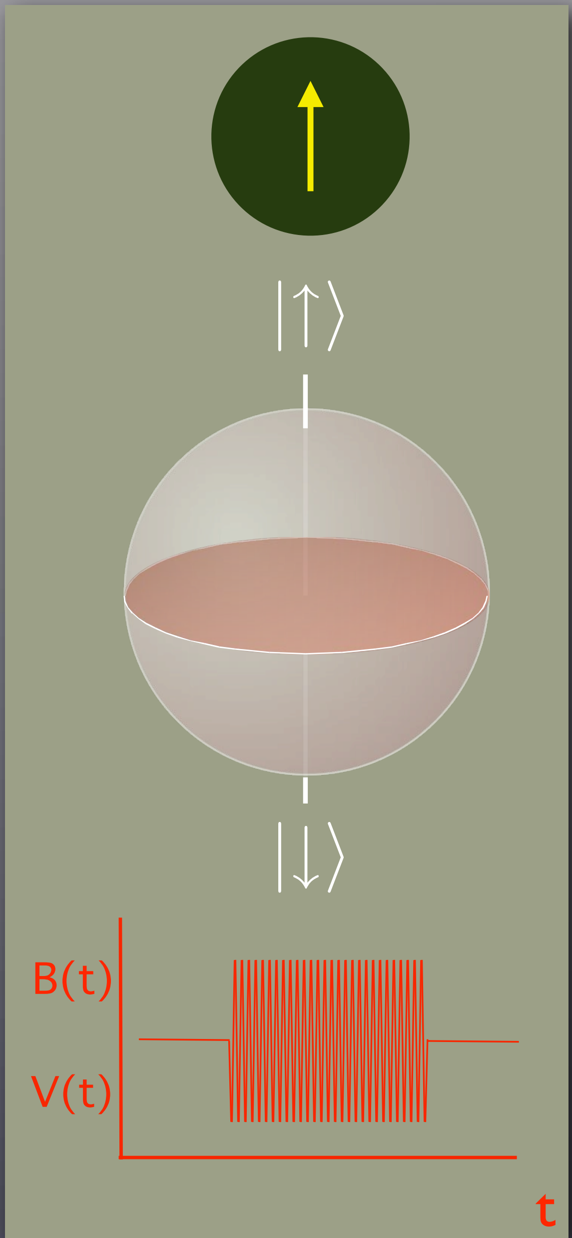


Two-Qubit Gate of Combined Single-Spin Rotation and Interdot Spin Exchange in a Double Quantum Dot

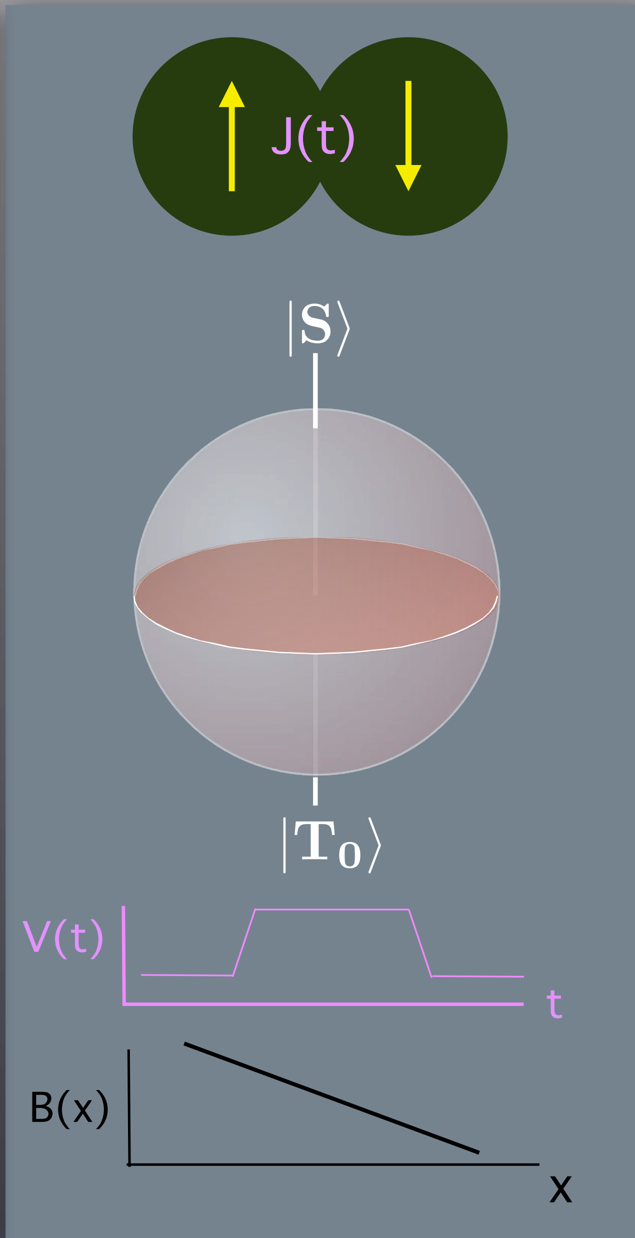
R. Brunner,[1,2,*] Y.-S. Shin,[1] T. Obata,[1,3] M. Pioro-Ladrière,[4] T. Kubo,[5] K. Yoshida,[1] T. Taniyama,[6,7]
Y. Tokura,[1,5] and S. Tarucha[1,3]



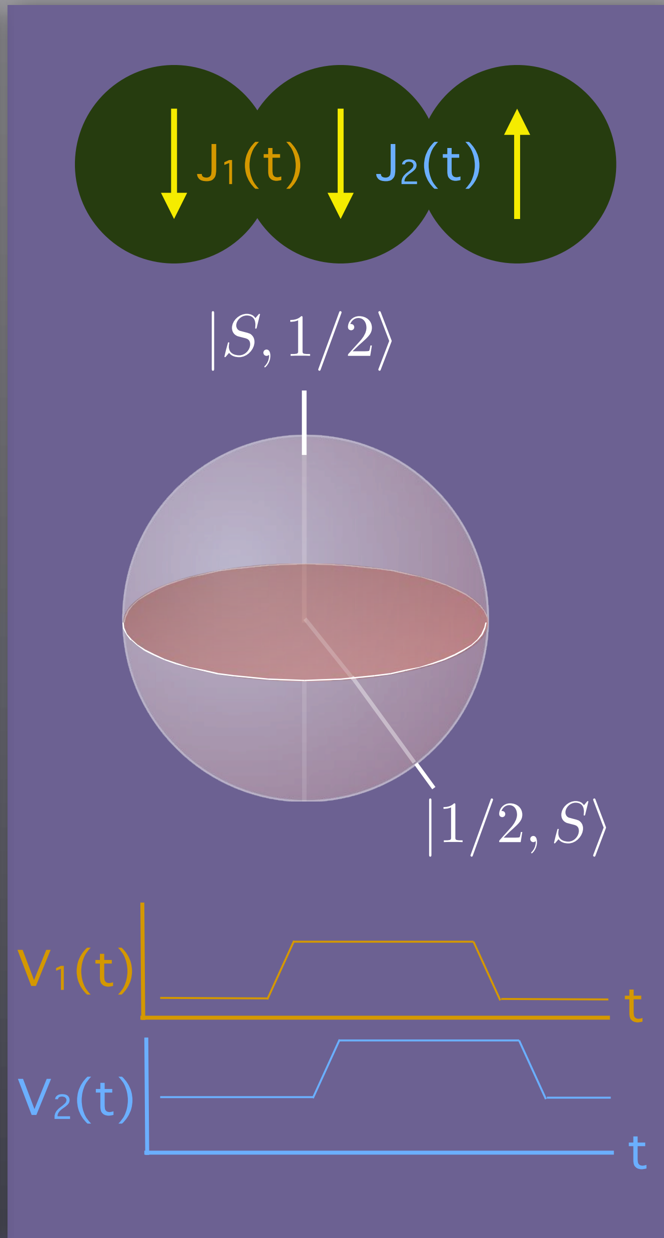
More Choices and Trade-offs for the Logical Qubit



Loss DiVincenzo, PRA 1998

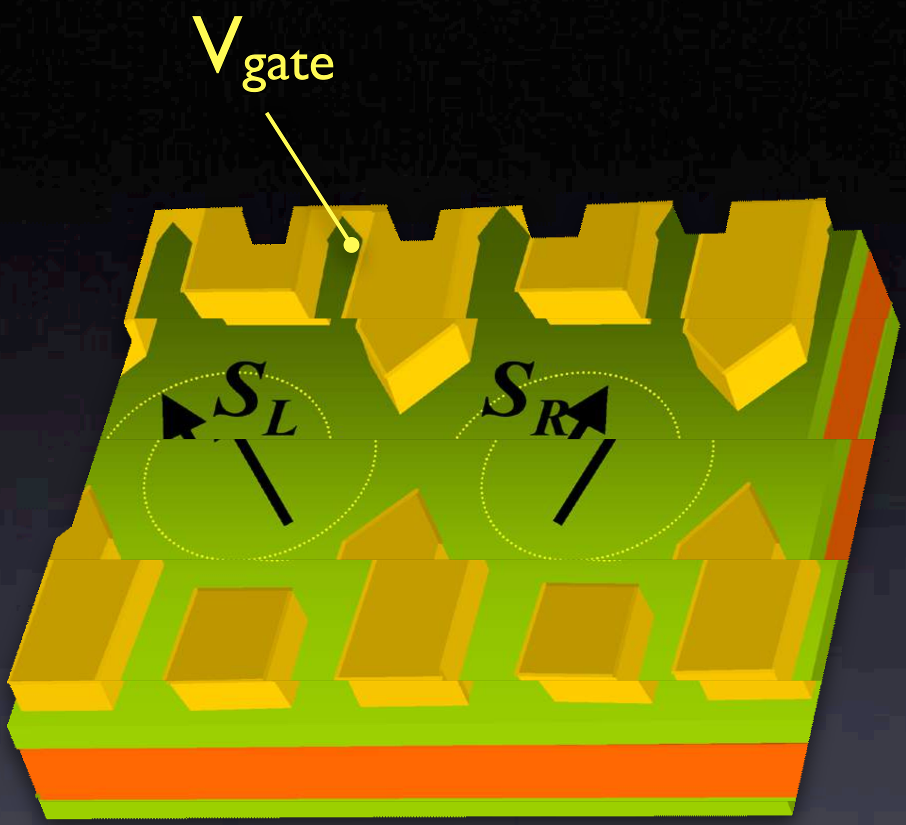


Levy, PRL 2002



DiVincenzo, et al, 2000, Laird, et al. 2010

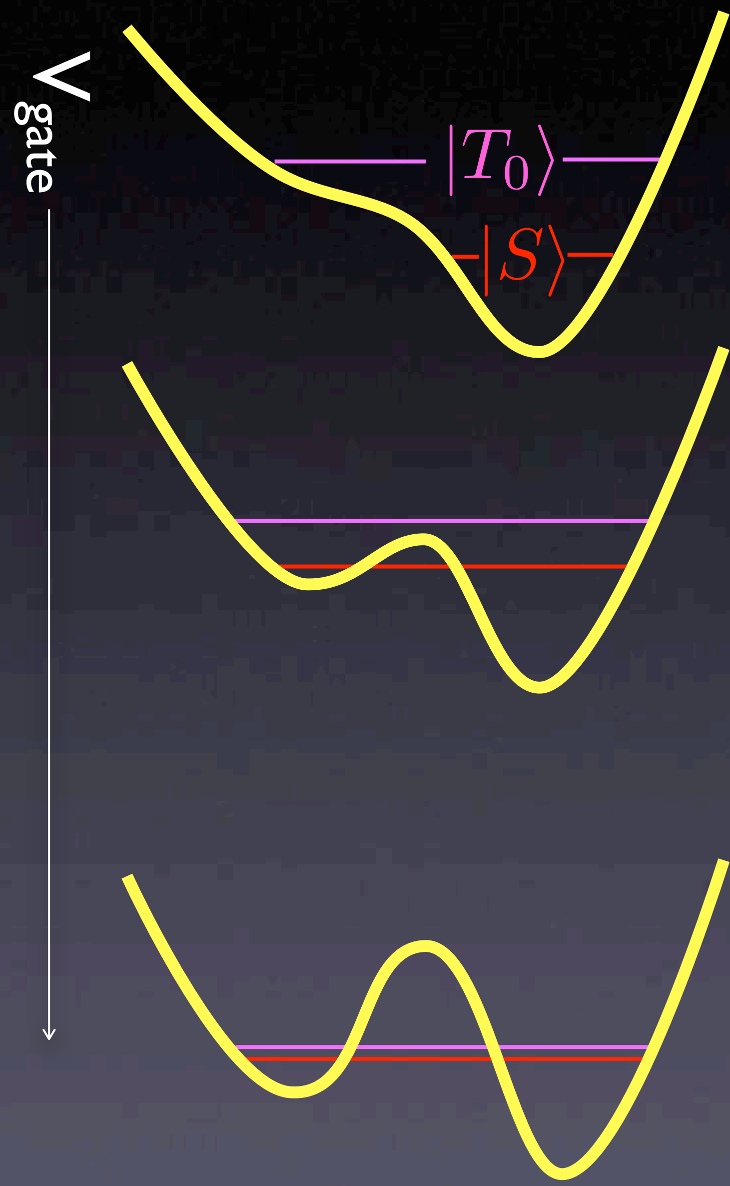
Gate control of the S-T qubit

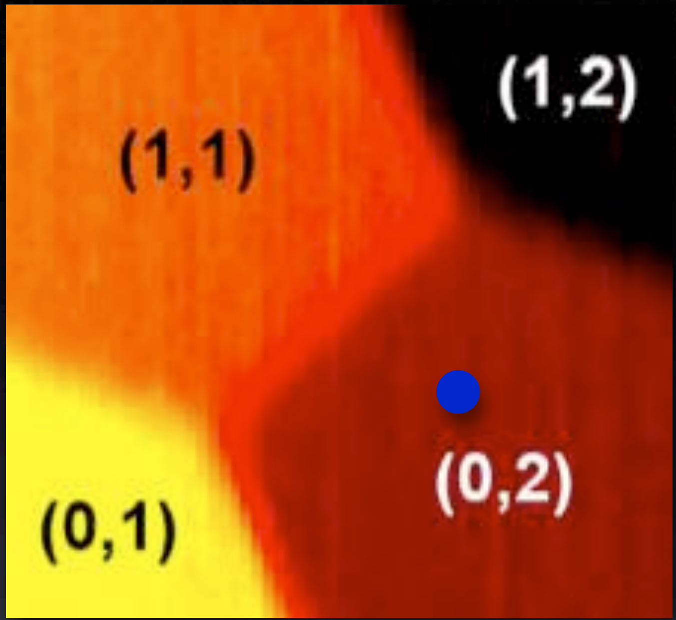


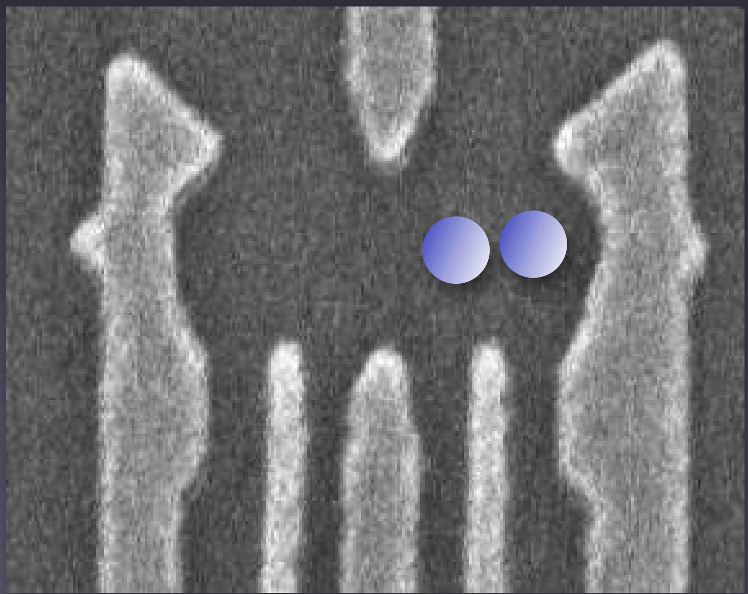
$$|S\rangle = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

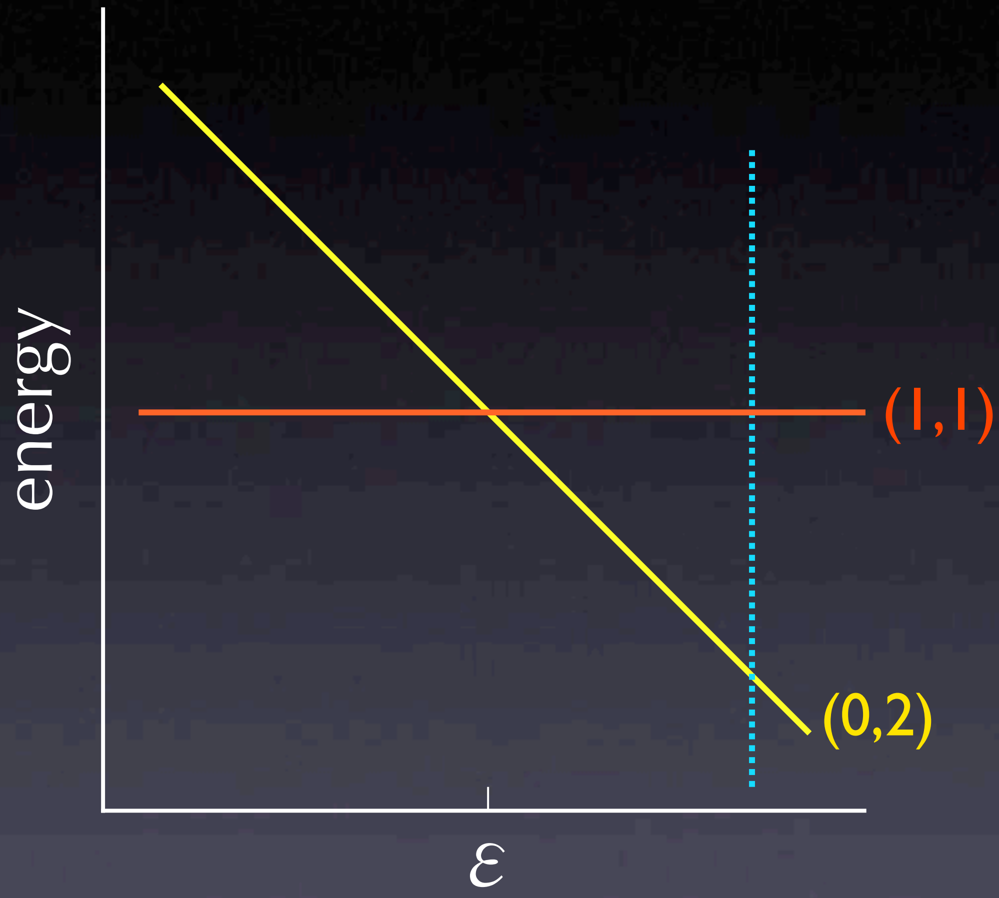
$$|T_0\rangle = |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$$

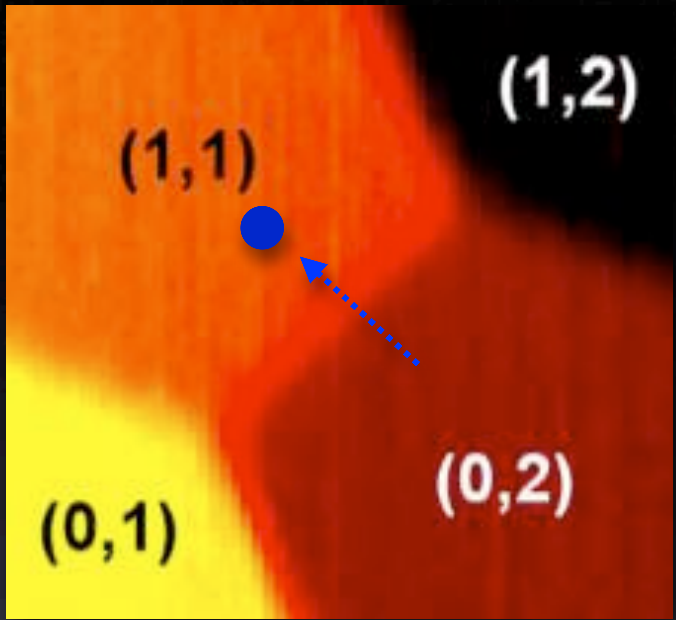
two-electron states

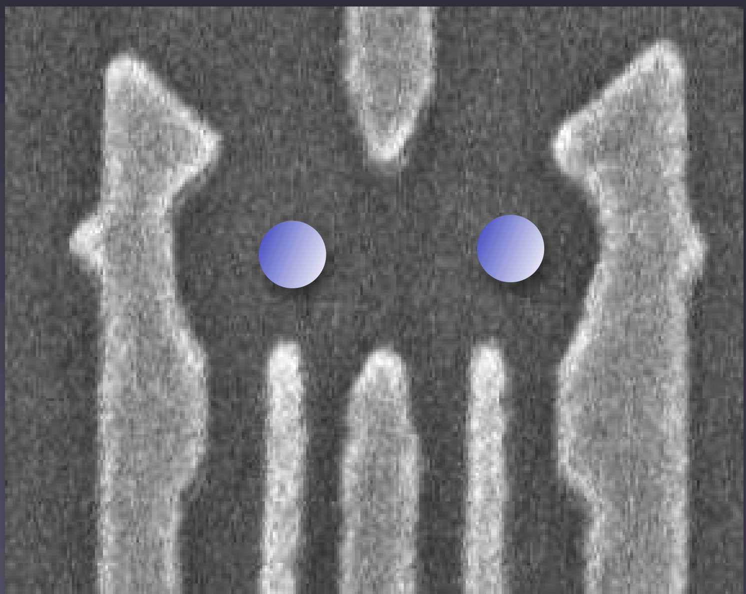


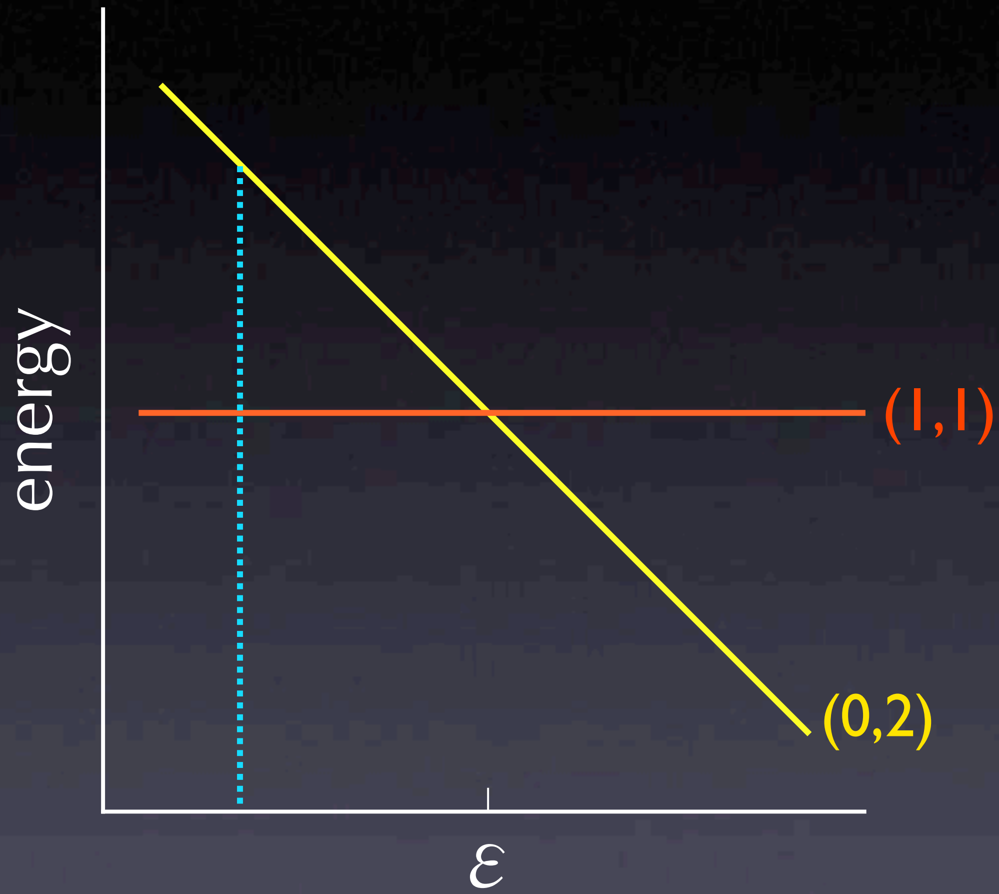

(1,2)
(1,1)
(0,2)
(0,1)

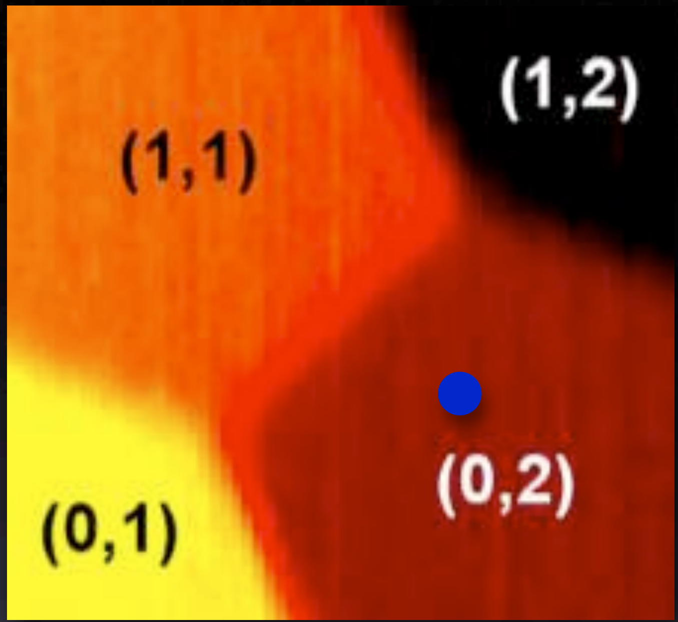


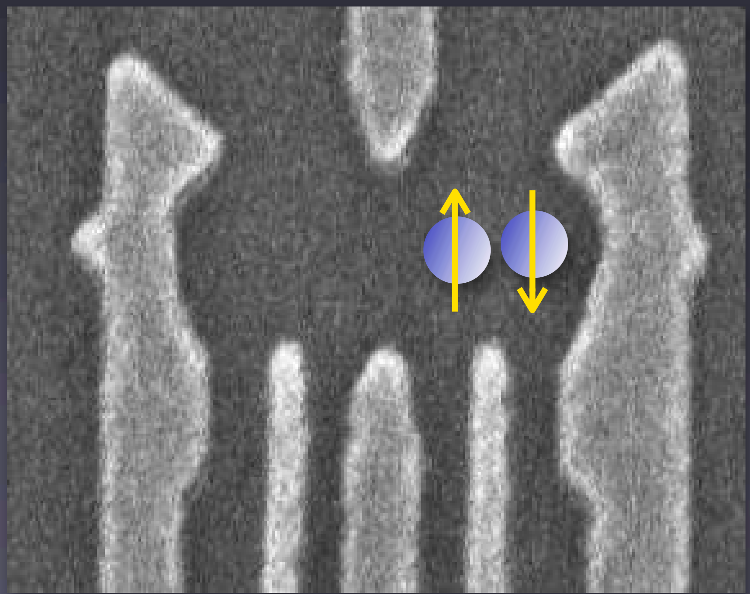

energy
(1,1)
(0,2)
ε

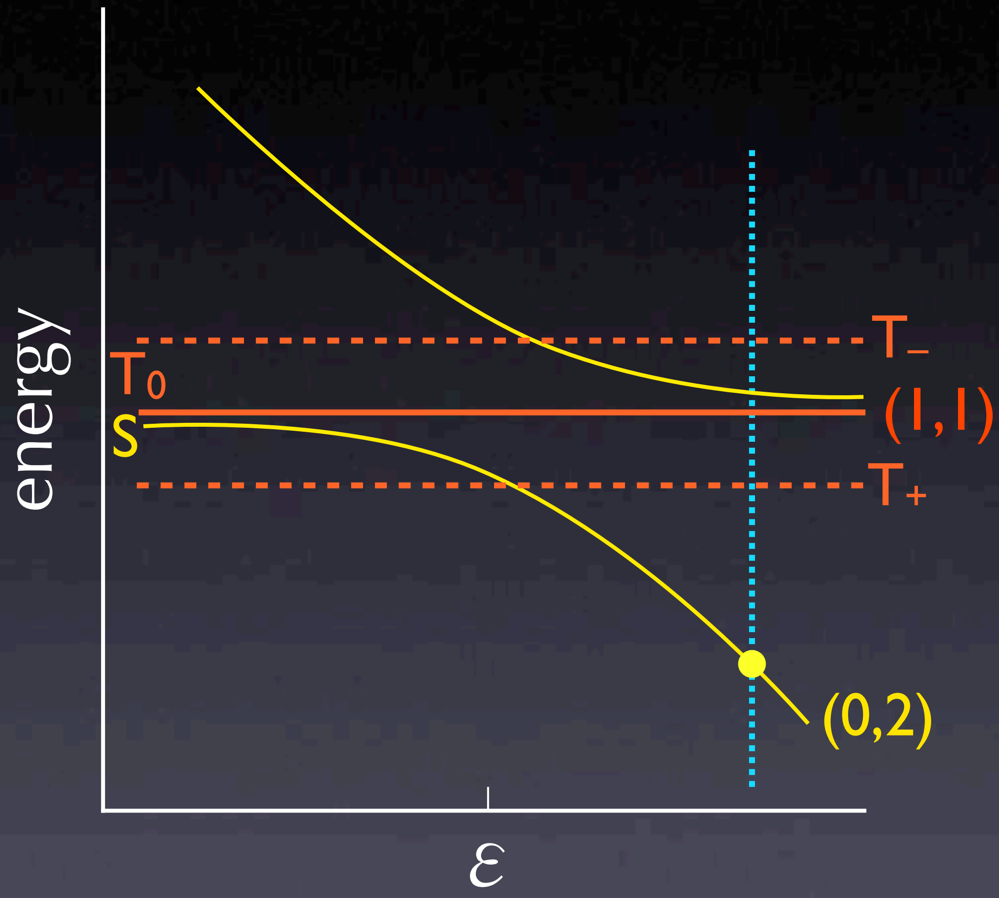

(1,2)
(1,1)
(0,2)
(0,1)

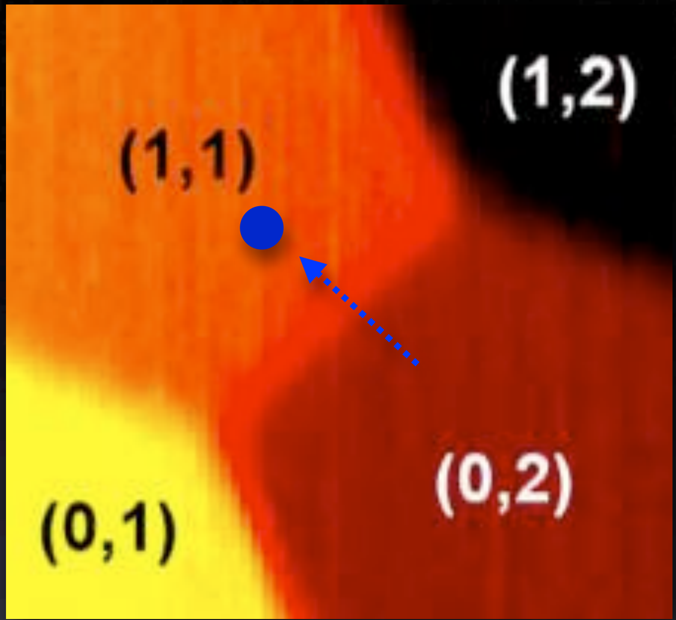


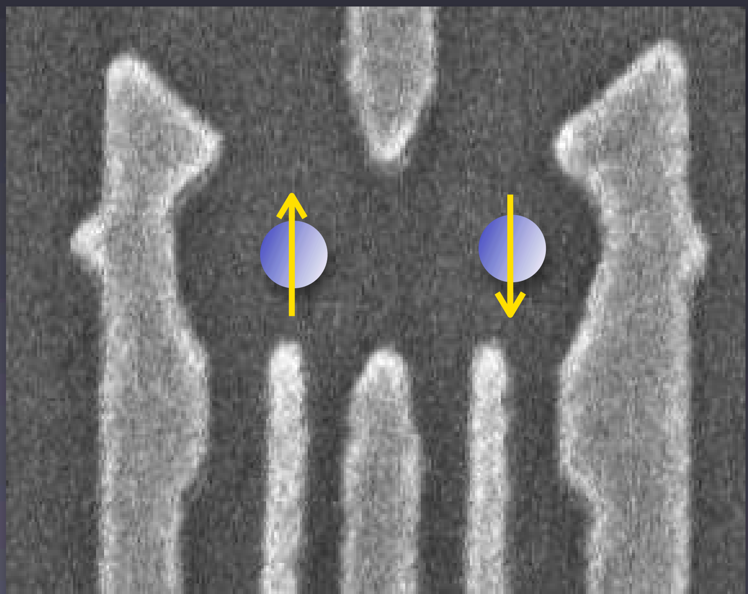

energy
(1,1)
(0,2)
ε

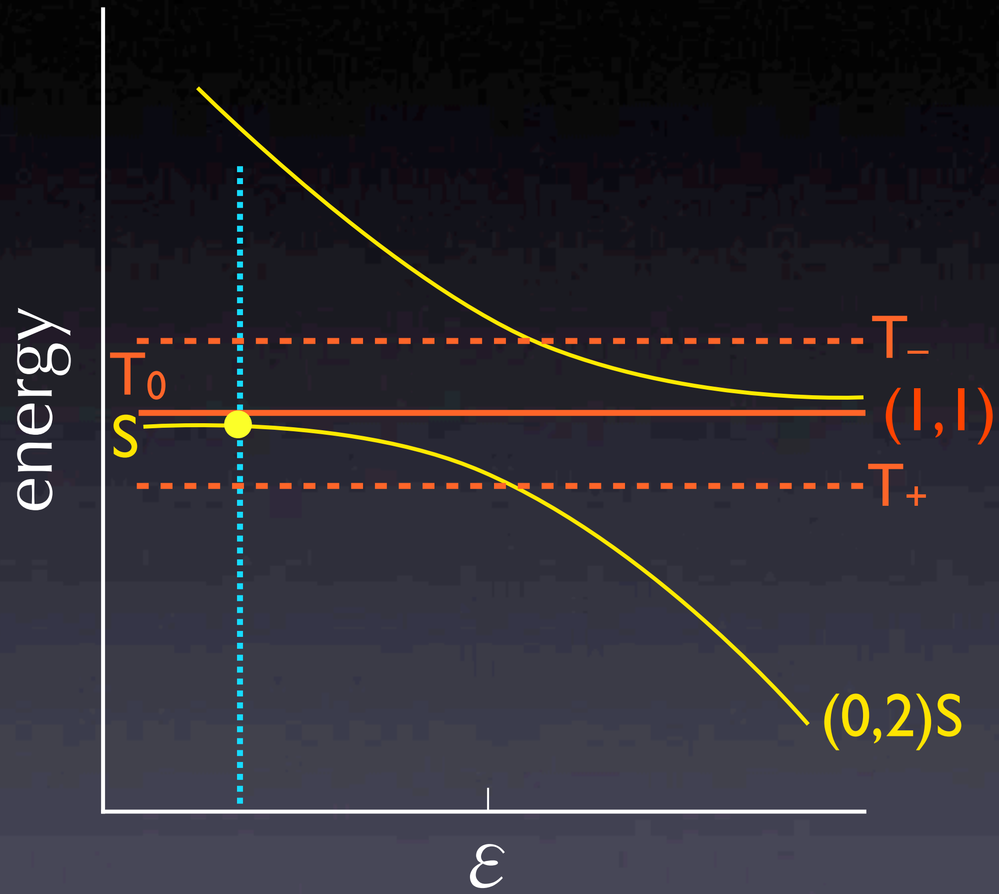

(1,2)
(1,1)
(0,2)
(0,1)

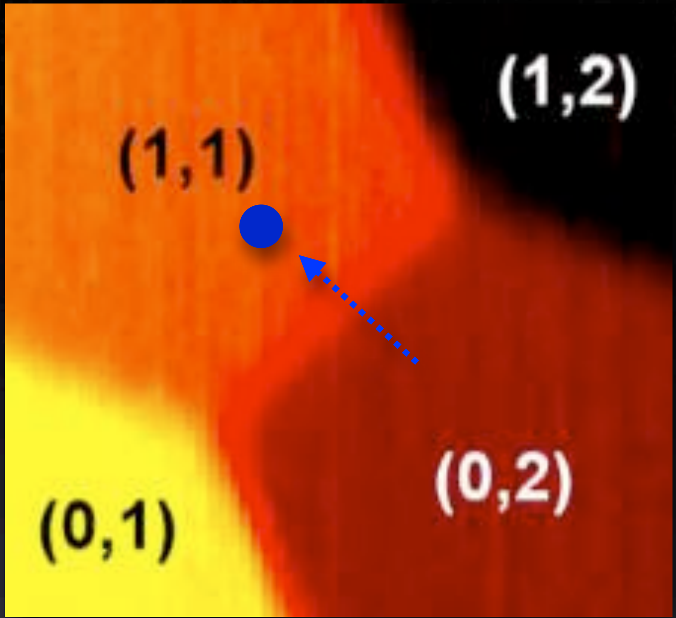


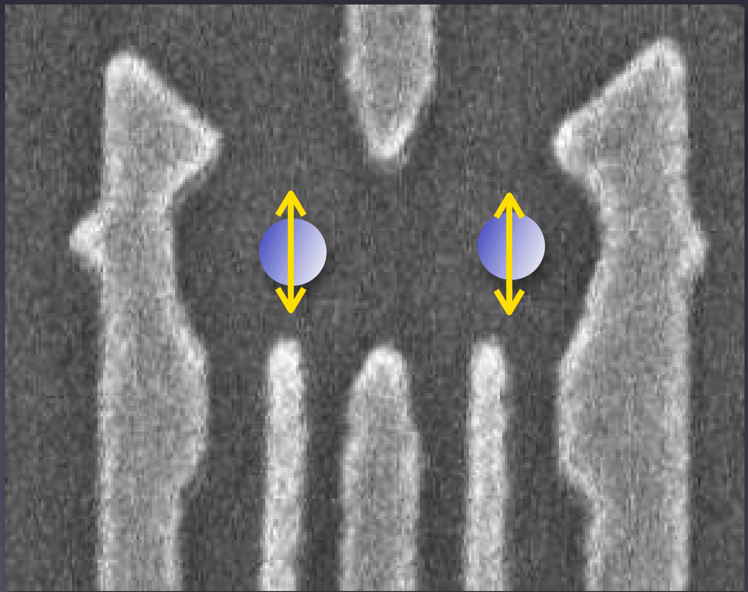

energy
T−
T0
S
(1,1)
T+
(0,2)
ε

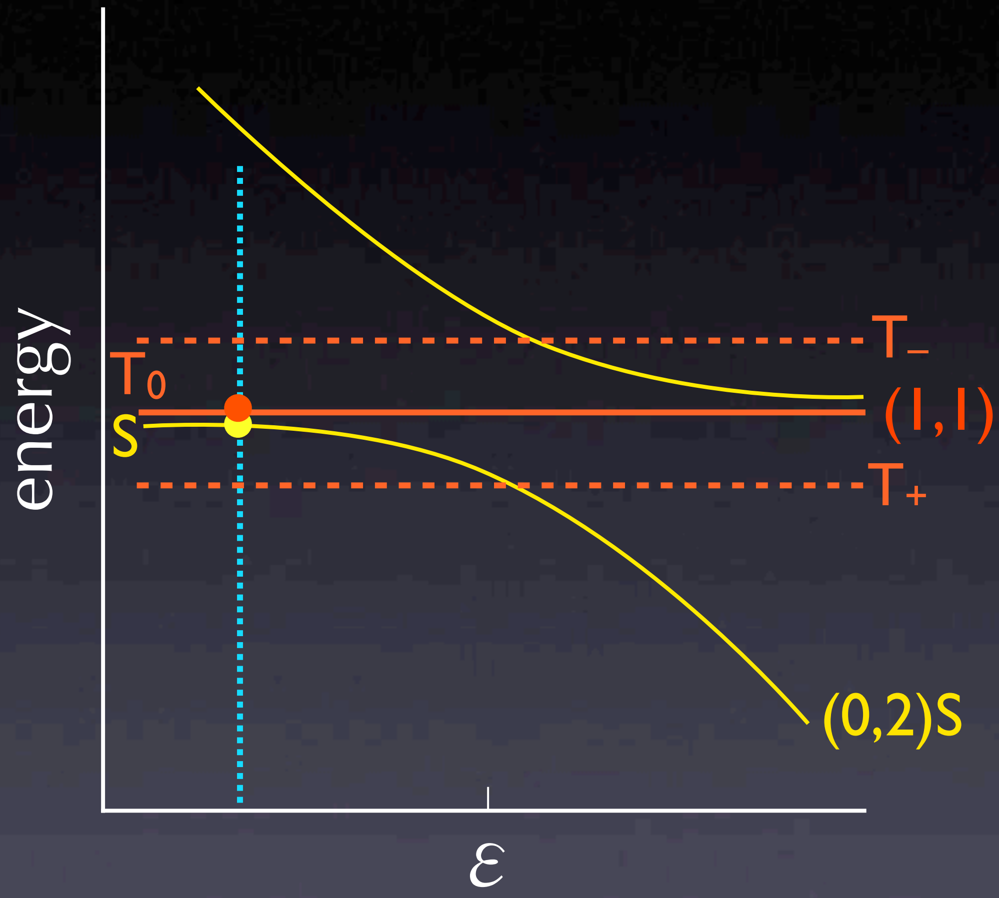

(1,2)
(1,1)
(0,2)
(0,1)

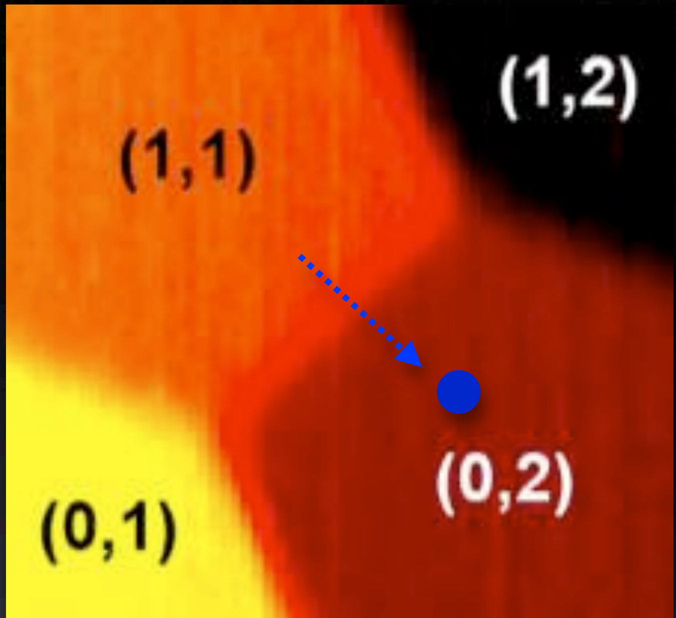


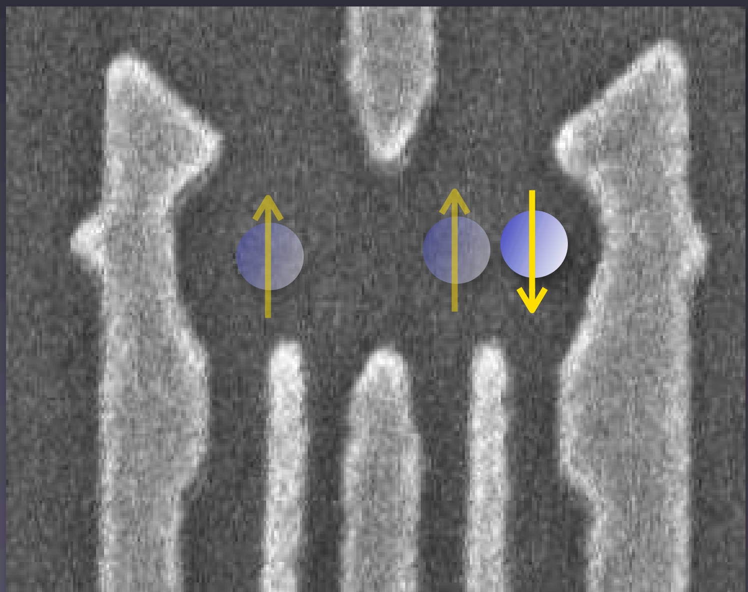

energy
T_-
T_0
S
(1,1)
T_+
(0,2)S
ε

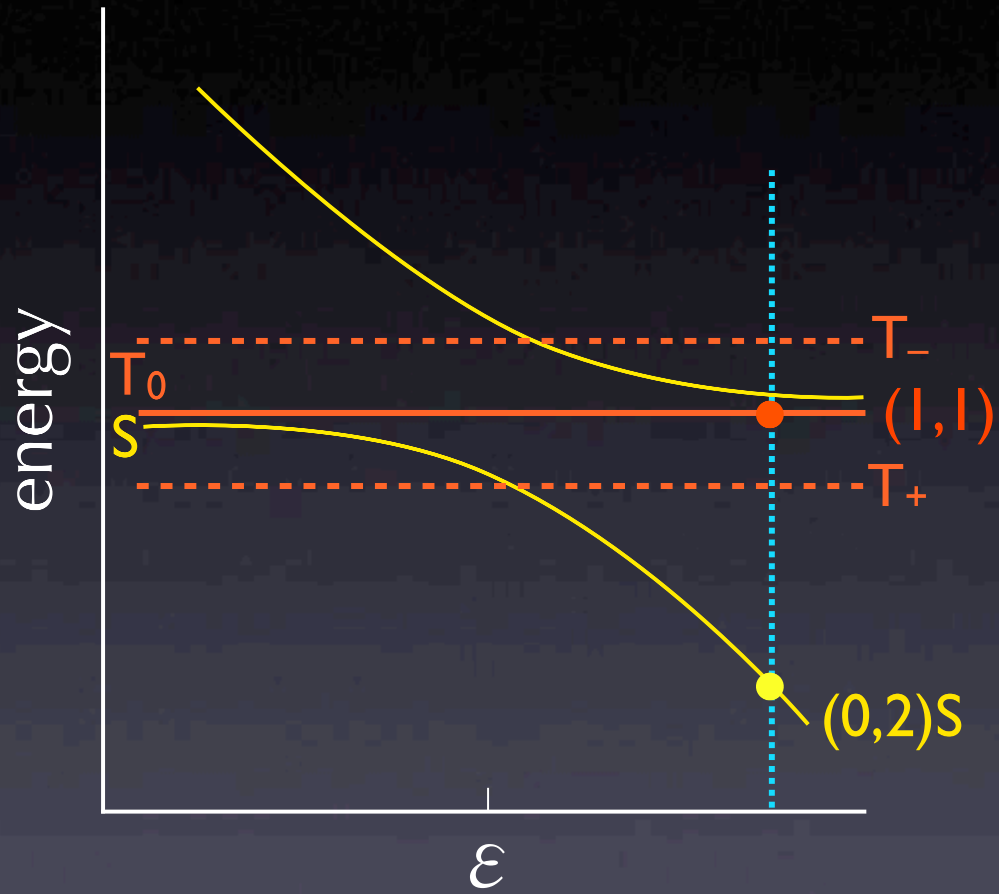

(1,2)
(1,1)
(0,2)
(0,1)



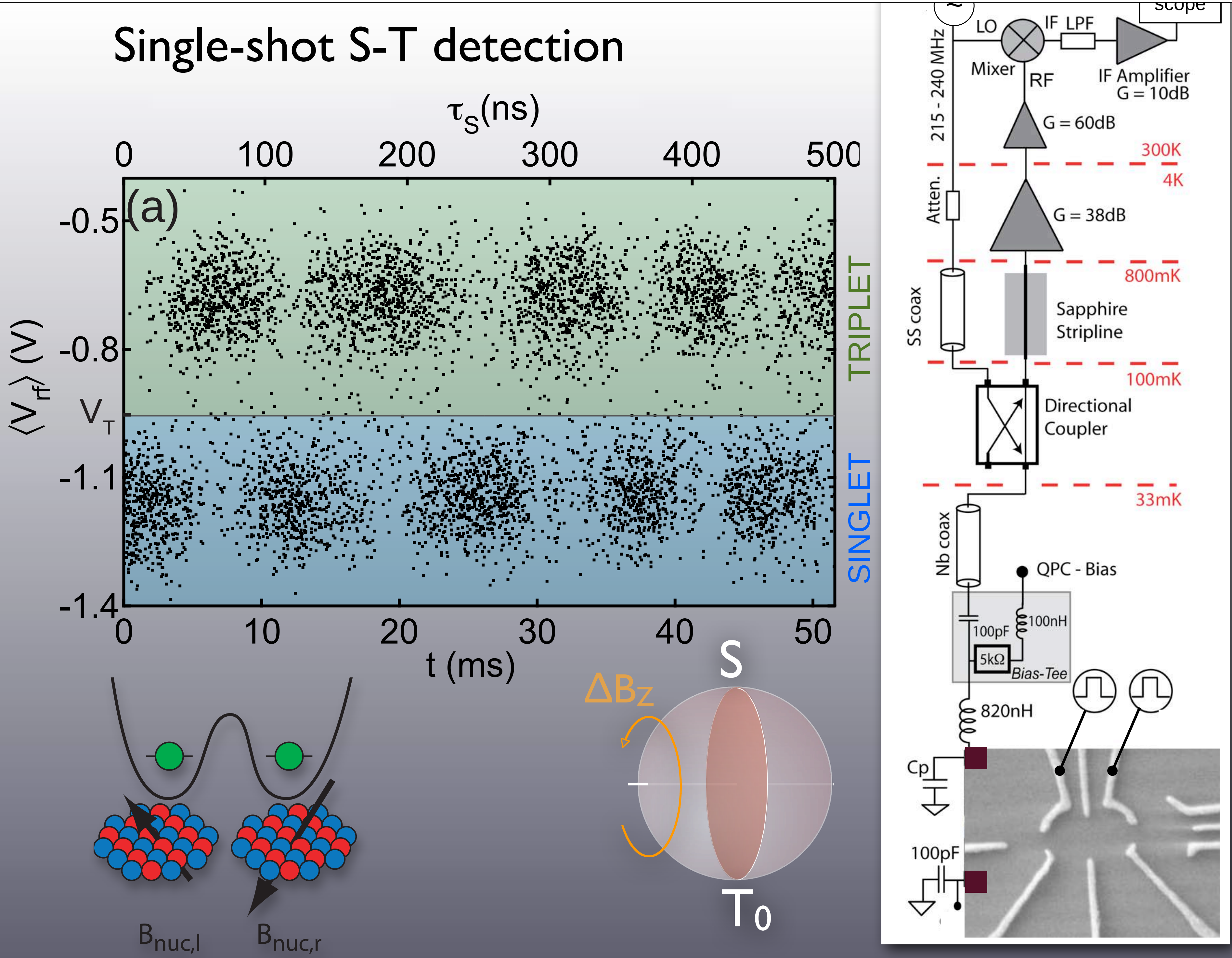

energy
T–
T0
(1,1)
S
T+
(0,2)S
ε


(1,2)
(1,1)
(0,2)
(0,1)




energy
T_-
T_0
(1,1)
S
T_+
(0,2)S
ε

Single-shot S-T detection



C. Barthel et al., Phys. Rev. Lett. 103, 160503 (2009).

Nuclear coupling well understood

VOLUME 88, NUMBER 18 PHYSICAL REVIEW LETTERS 6 MAY 2002

Electron Spin Decoherence in Quantum Dots due to Interaction with Nuclei

Alexander V. Khaetskii,[1] Daniel Loss,[1] and Leonid Glazman[2]
[1]*Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland*
[2]*Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455*
(Received 17 January 2002; published 19 April 2002)

We study the decoherence of a single electron spin in an isolated quantum dot induced by hyperfine interaction with nuclei. The decay is caused by the spatial variation of the electron wave function within the dot, leading to a nonuniform hyperfine coupling A. We evaluate the spin correlation function and find that the decay is not exponential but rather power (inverse logarithm) lawlike. For polarized nuclei we find an exact solution and show that the precession amplitude and the decay behavior can be tuned by the magnetic field. The decay time is given by $\hbar N/A$, where N is the number of nuclei inside the dot, and the amplitude of precession decays to a finite value. We show that there is a striking difference between the decoherence time for a single dot and the dephasing time for an ensemble of dots.

DOI: 10.1103/PhysRevLett.88.186802 PACS numbers: 73.21.La, 76.20.+q, 76.60.Es, 85.35.Be

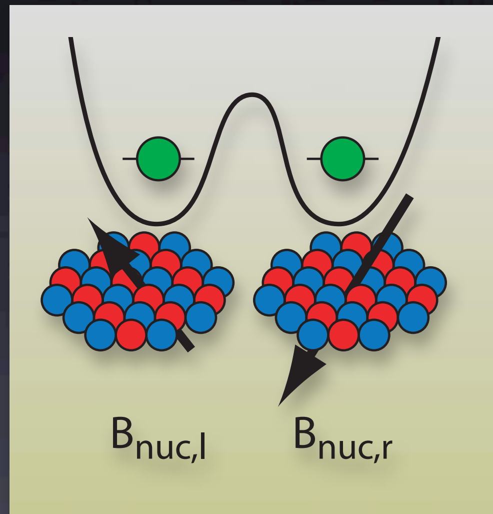
The spin dynamics of electrons in semiconducting nanostructures has become of central interest in recent years [1]. The controlled manipulation of spin, and in particular of its phase, is the primary prerequisite needed for novel applications in conventional computer hardware as well as in quantum information processing. It is thus desirable to unders
the spin phase coheren
GaAs semiconductors, v
unusually long spin d
100 ns [2]. Since in Ga
hyperfine interaction be
is unavoidable, and it is
its effect on the electr
particularly so for electr
system such as a quant
state, since, besides fu
are promising candidate
recent work on spin rela
in GaAs nanostructures
Motivated by this we
spin dynamics of a single electron confined to a quantum dot in the presence of nuclear spins. We treat the case of unpolarized nuclei perturbatively, while for the fully polarized case we present an exact solution for the spin dynamics and show that the decay is nonexponential and can be strongly influenced by external magnetic fields. We use the term "decoherence" to describe the case with a single dot, and the term "dephasing" for an ensemble of dots [8]. The typical fluctuating nuclear magnetic field seen by the electron spin via the hyperfine interaction is of the order of [9] $\sim A/\sqrt{N}\, g\mu_B$, with an associated electron precession frequency $\omega_N \simeq A/\sqrt{N}$, where A is a hyperfine constant, g the electron g factor, and μ_B the Bohr magneton. For a typical dot size the electron wave function covers approximately $N = 10^5$ nuclei, then this field is of the order of 100 G in a GaAs quantum dot. The nuclei in

turn interact with each other via dipolar interaction, which does not conserve the total nuclear spin and thus leads to a change of a given nuclear spin configuration within the time $T_{n2} \approx 10^{-4}$ s, which is just the period of precession of a nuclear spin in the local magnetic field generated by its neighbors.

i.e., $T_{n2} \to \infty$, no averaging is indicated. However, each flip-flop process (due to hyperfine interaction) creates a different nuclear configuration, and because of the spatial variation of the hyperfine coupling constants inside the dot, this leads to a different value of the nuclear field seen by the electron spin and thus to its decoherence. Below we will find that this decoherence is nonexponential, but still we can indicate a characteristic time given by $(A/\hbar N)^{-1}$ [8]. Moreover, we shall find that $T_{n2} \gg (A/\hbar N)^{-1}$, and thus still no averaging over the nuclear configurations is indicated (and dipolar interactions will be neglected henceforth). To underline the importance of this point, we will contrast below the unaveraged correlator with its average.

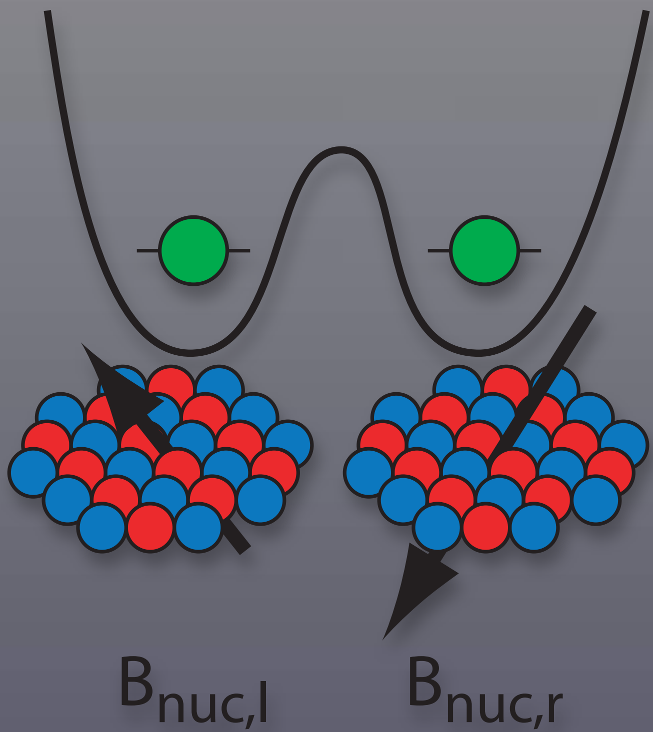
Unpolarized nuclei.—We consider a single electron confined to a quantum dot whose spin $\mathbf{S}$ couples to an external magnetic field $\mathbf{B}$ and to nuclear spins $\{\mathbf{I}^i\}$ via

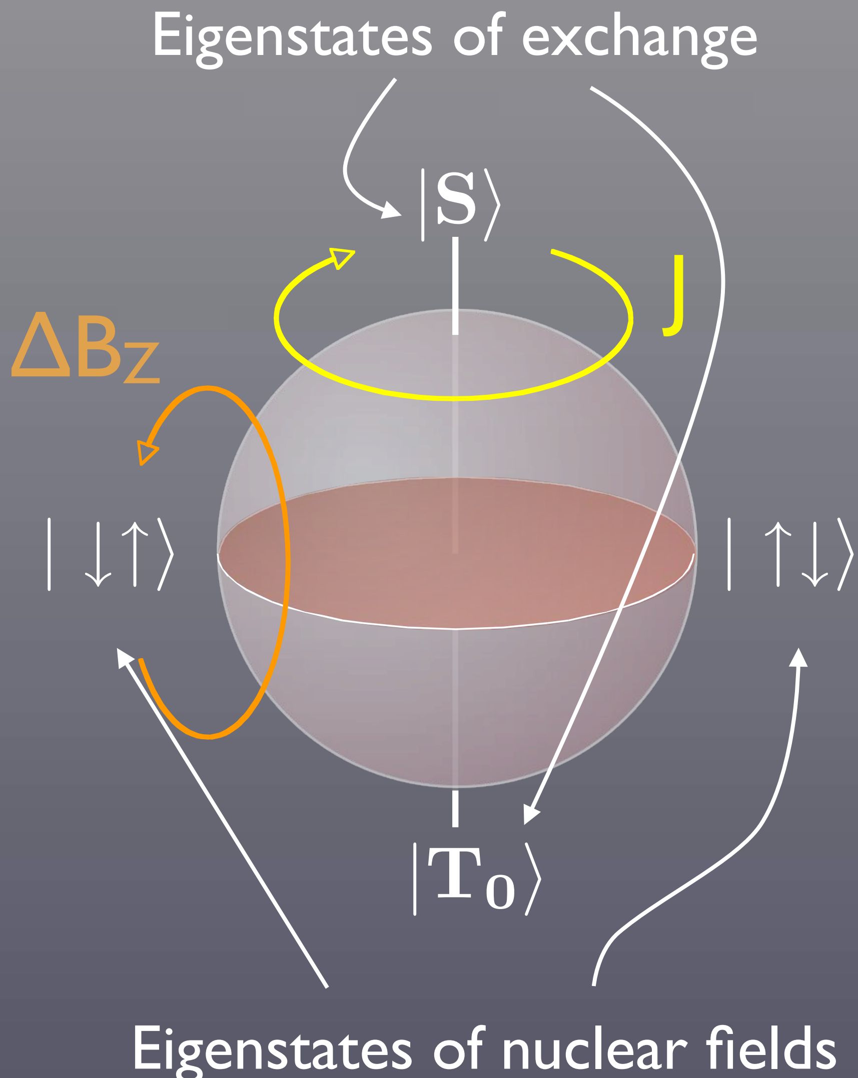
The typical fluctuating nuclear magnetic field seen by the electron spin via the hyperfine interaction is of the order of [9] $\sim A/\sqrt{N}\, g\mu_B$, with an associated electron precession frequency $\omega_N \simeq A/\sqrt{N}$, where A is a hyperfine constant, g the electron g factor, and μ_B the Bohr magneton. For a typical dot size the electron wave function covers approximately $N = 10^5$ nuclei, then this field is of the order of 100 G in a GaAs quantum dot. The nuclei in



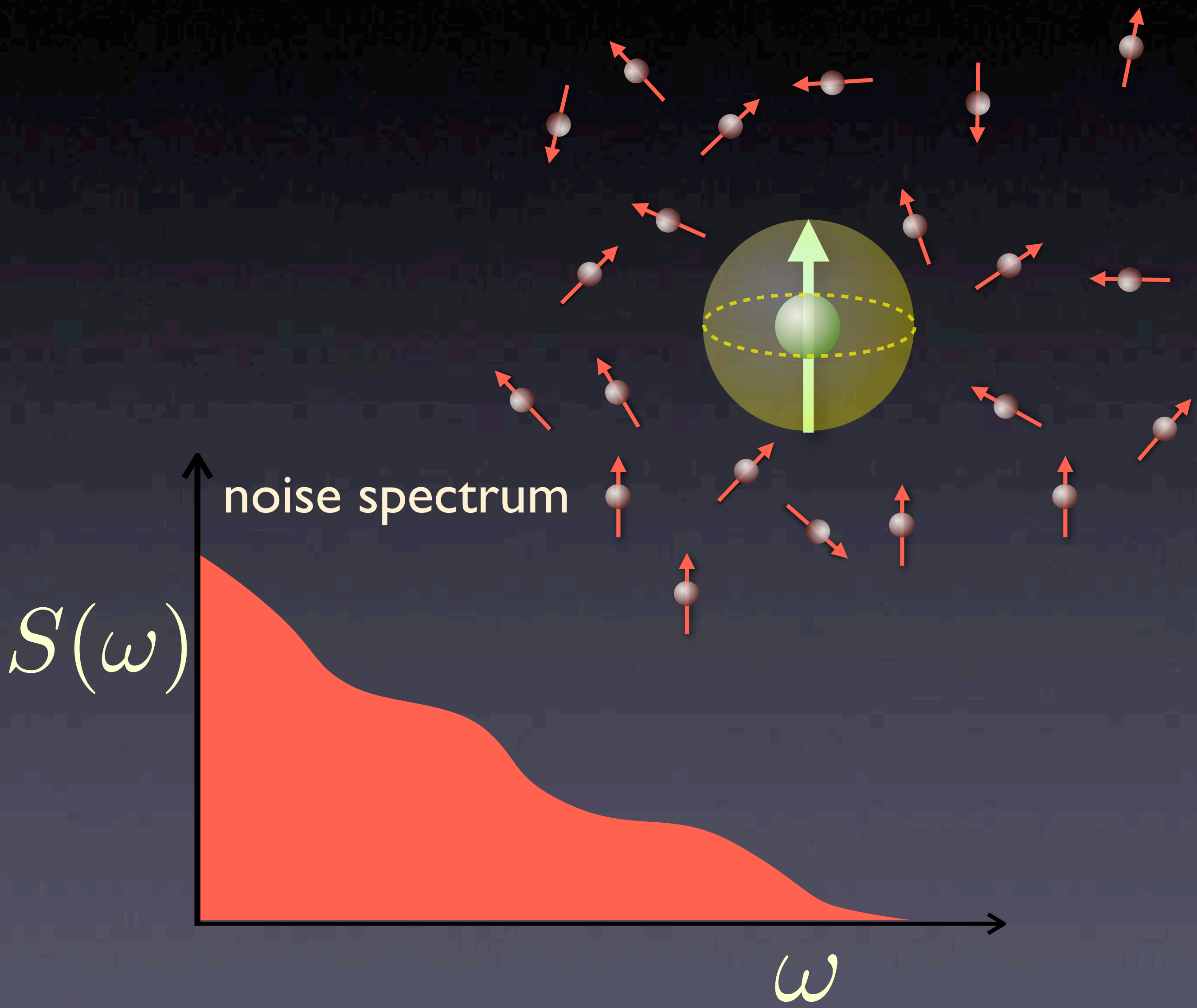
$$T2^* \sim \omega_N^{-1} \sim 10ns$$

Hyperfine coupling and the S-T qubit



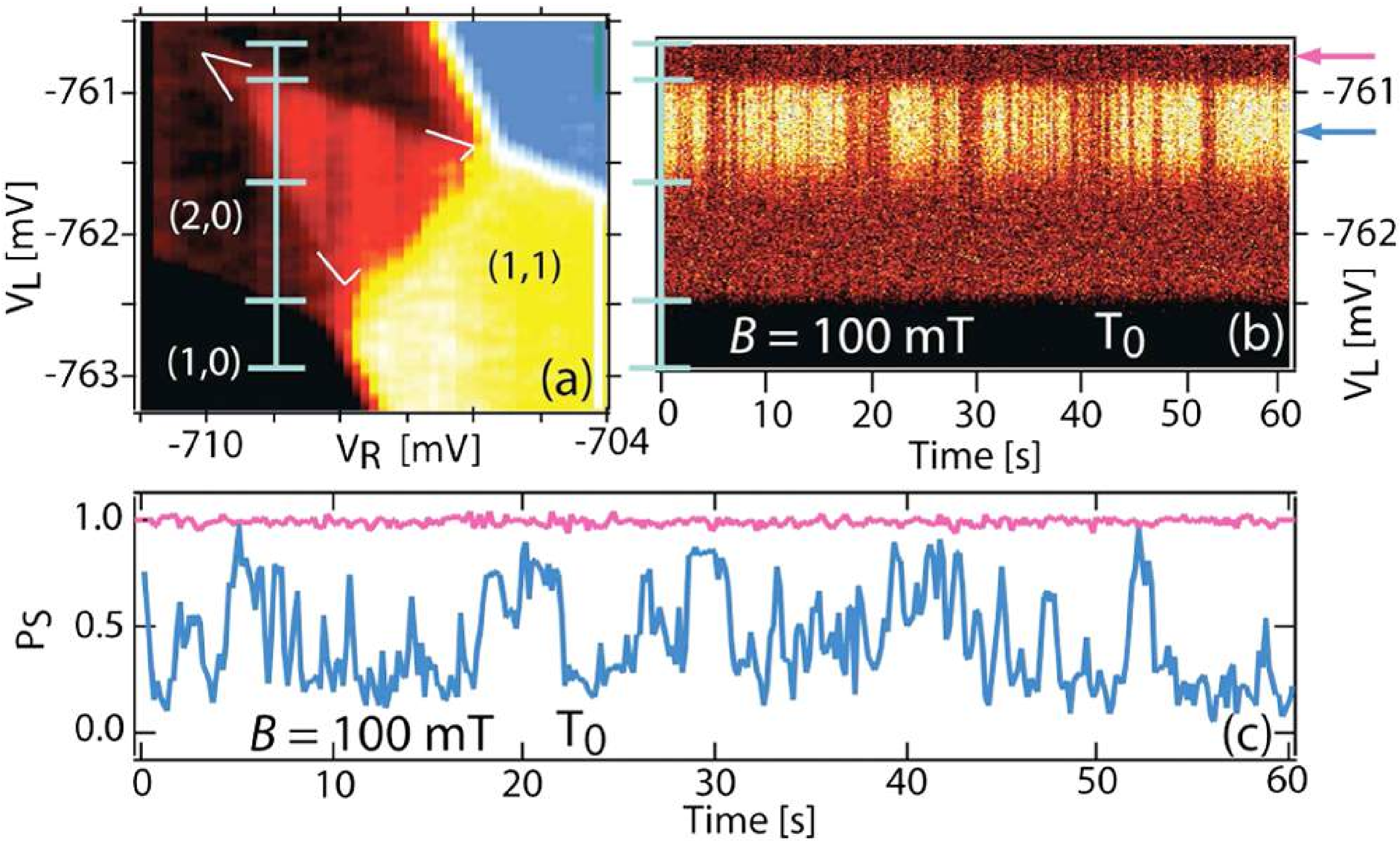


Spin Qubit



Measurement of Temporal Correlations of the Overhauser Field in a Double Quantum Dot

D. J. Reilly,[1,*] J. M. Taylor,[2] E. A. Laird,[1] J. R. Petta,[3] C. M. Marcus,[1] M. P. Hanson,[4] and A. C. Gossard[4]
[1]*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[2]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[3]*Department of Physics, Princeton University, Princeton, New Jersey 08544, USA*
[4]*Department of Materials, University of California, Santa Barbara, California 93106, USA*
(Received 24 December 2007; published 4 December 2008)



PRL 101, 236803 (2008) PHYSICAL REVIEW LETTERS week ending 5 DECEMBER 2008

Measurement of Temporal Correlations of the Overhauser Field in a Double Quantum Dot

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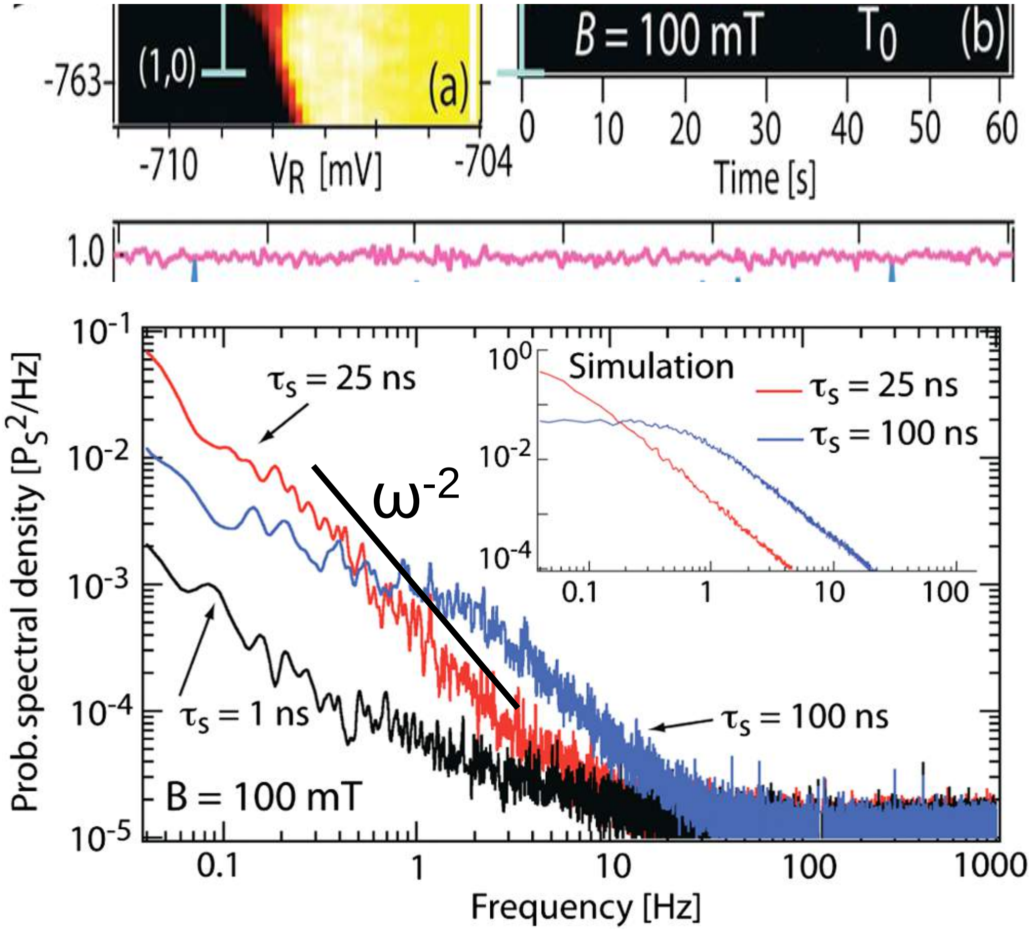
[1]Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA
[2]Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
[3]Department of Physics, Princeton University, Princeton, New Jersey 08544, USA
[4]Department of Materials, University of California, Santa Barbara, California 93106, USA
(Received 24 December 2007; published 4 December 2008)



Model spectral diffusion

$$C(t) = \langle B_{nuc}(t) B_{nuc}(0) \rangle$$
$$\propto e^{(-t/\tau_C)}$$

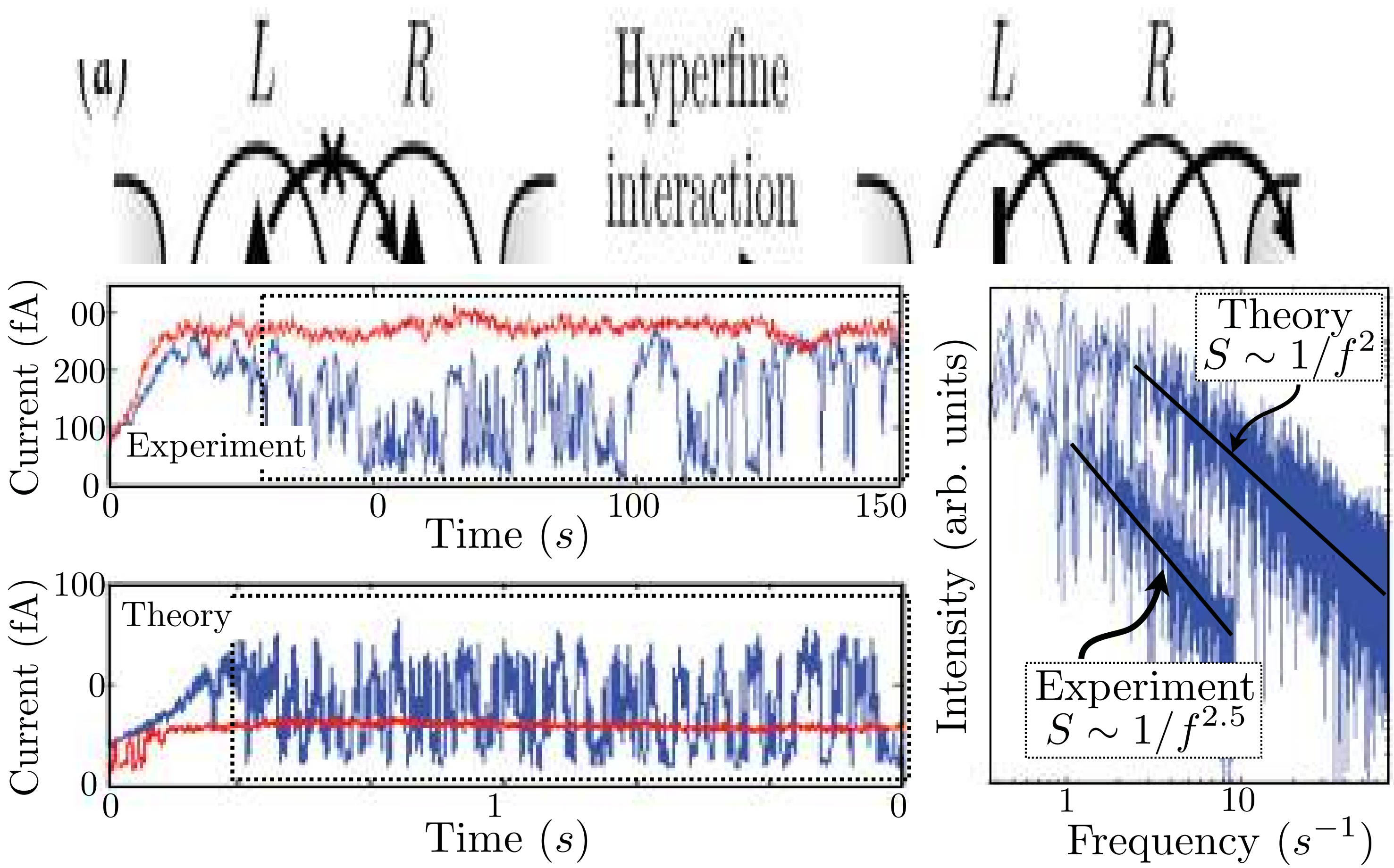
Nuclear spectral function

$$S(\omega) = A\,\omega^{-\beta}$$
$$\beta = 2$$

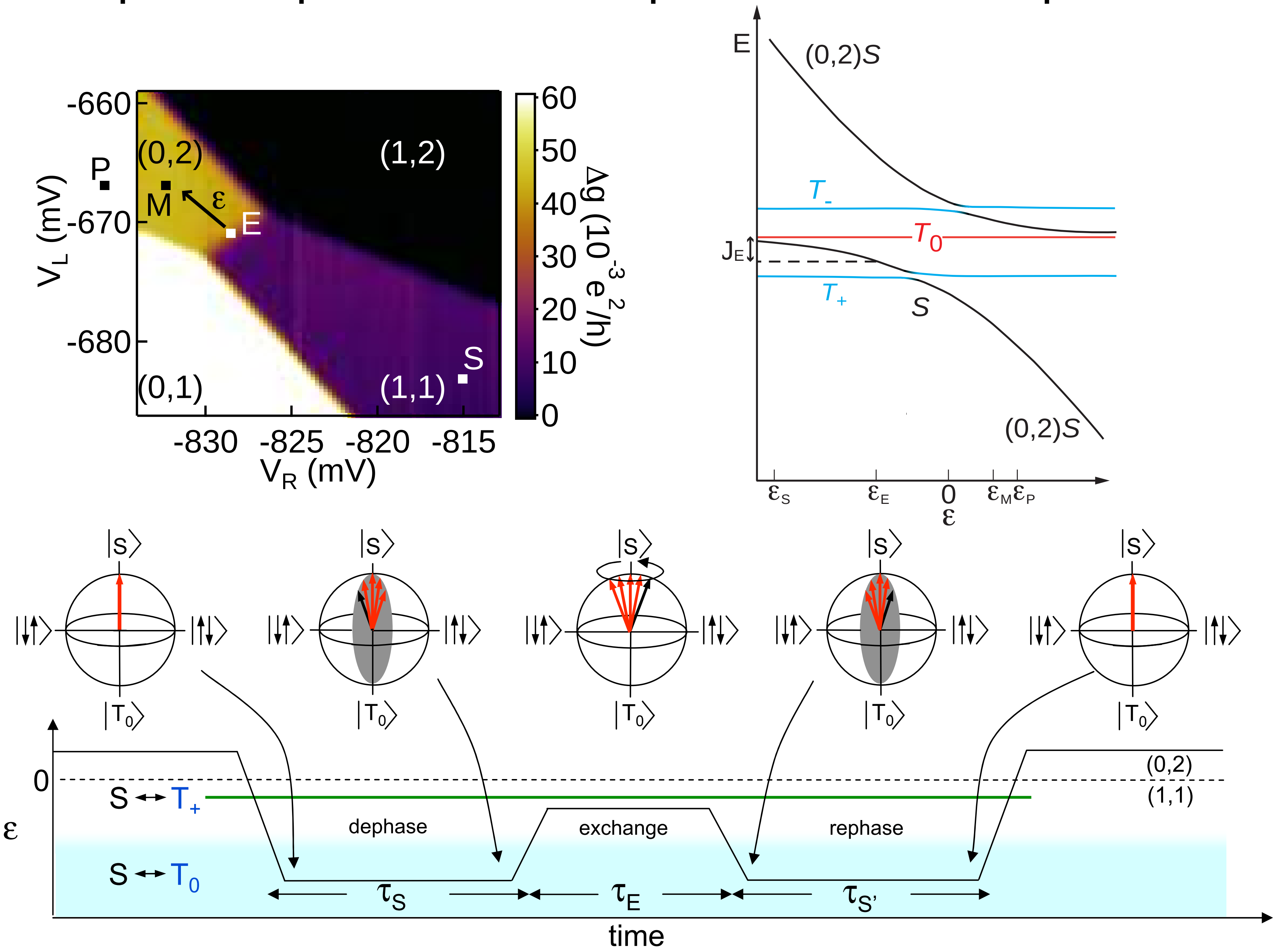
PHYSICAL REVIEW B **84**, 075339 (2011)

Nuclear spin dynamics in double quantum dots: Fixed points, transients, and intermittency

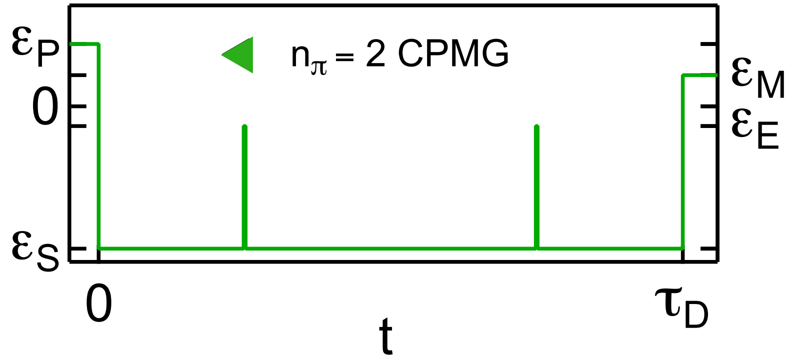
M. S. Rudner,[1] F. H. L. Koppens,[2,3] J. A. Folk,[2,4] L. M. K. Vandersypen,[2] and L. S. Levitov[5]
[1]*Department of Physics, Harvard University, 17 Oxford Street, Cambridge, Massachusetts 02138, USA*
[2]*Kavli Institute of NanoScience, TU Delft, P.O. Box 5046, NL-2600 GA, Delft, The Netherlands*
[3]*ICFO–Institut de Ciencies Fotoniques, Mediterranean Technology Park, E-08860 Castelldefels (Barcelona), Spain*
[4]*Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z4*
[5]*Department of Physics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA*
(Received 8 July 2011; published 17 August 2011)

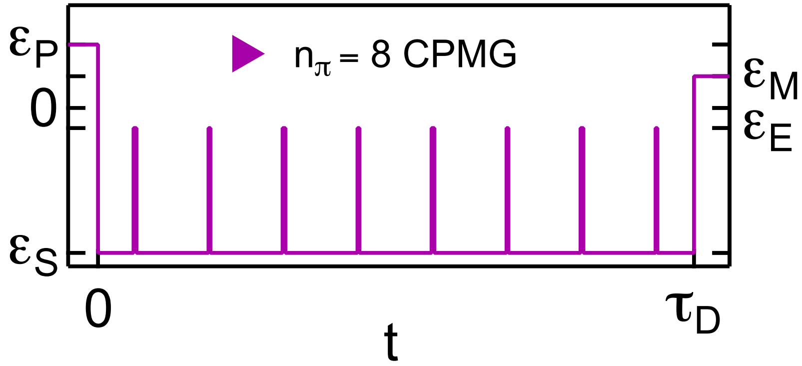


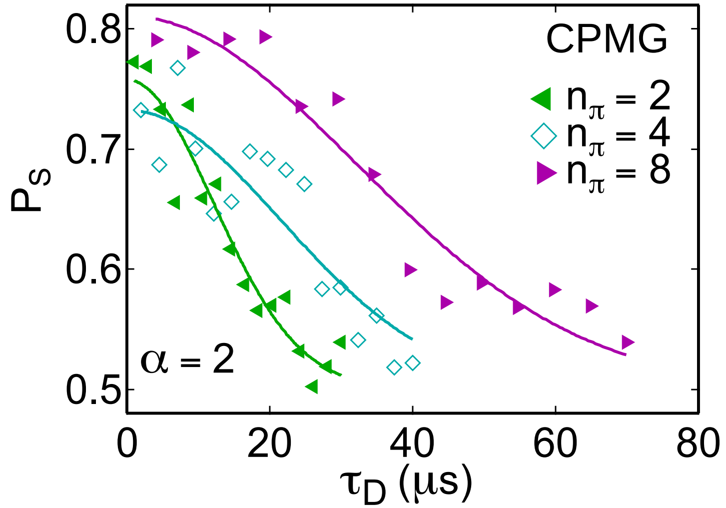
Prepare-Separate-Echo-Separate-Return sequence

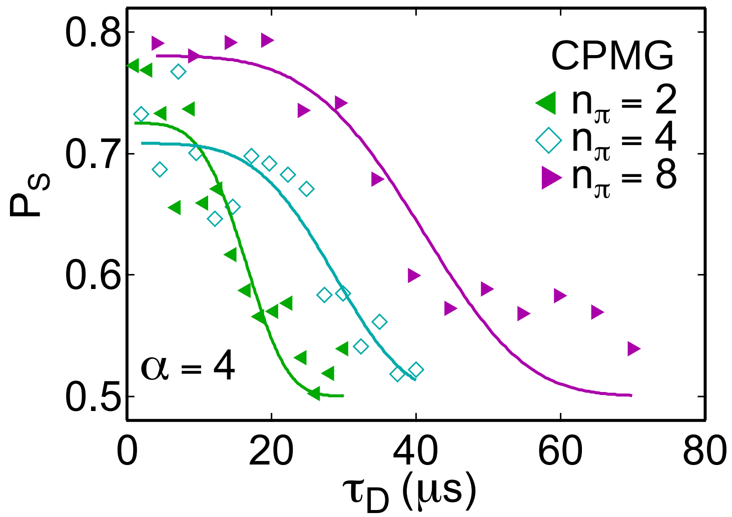


From Echo to CPMG









Envelope function for singlet return probability

$$P_S(\tau_D) = 0.5 + Ve^{-\left(\frac{\tau_D}{T_2}\right)^{\alpha}}$$

Different models give different values of α

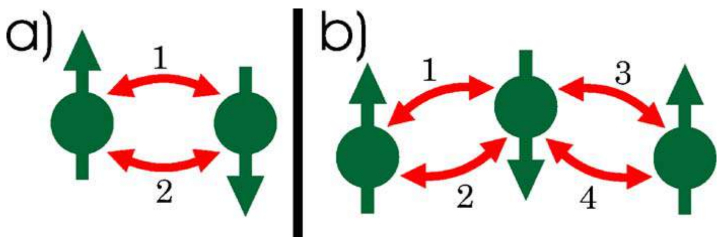
Difficult to measure α directly (compare α=2 and α =4 fits).

Theory: Cywinski et al.. Physical Review B **77** 174509 (2008)

PHYSICAL REVIEW B **74**, 035322 (2006)

Quantum theory for electron spin decoherence induced by nuclear spin dynamics in semiconductor quantum computer architectures: Spectral diffusion of localized electron spins in the nuclear solid-state environment

W. M. Witzel and S. Das Sarma
Condensed Matter Theory Center, Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA
(Received 14 December 2005; revised manuscript received 5 June 2006; published 18 July 2006)



The lowest order results, $v_E(\tau) \approx \exp[\langle \Sigma_2(\tau) \rangle]$, for most of our GaAs calculations show a Hahn echo decay of the form $\exp[-(2\tau/t_0)^4]$. This differs qualitatively from the decay for the Si:P which, by our calculations, is in the form exp $[-(2\tau/t_0)^\alpha]$ where $\alpha \sim 2.3$ for a range of τ appropriate for natural Si and some range of isotopic purification. The form of the GaAs echo decay does not change if we repeat the calculation with $I=1/2$ rather than $3/2$, although t_0 does change.

PHYSICAL REVIEW B **74**, 035322 (2006)

Quantum theory for electron spin decoherence induced by nuclear spin dynamics in semiconductor quantum computer architectures: Spectral diffusion of localized electron spins in the nuclear solid-state environment

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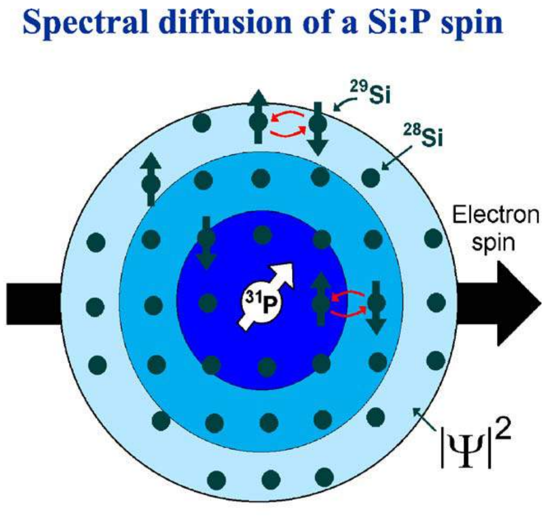
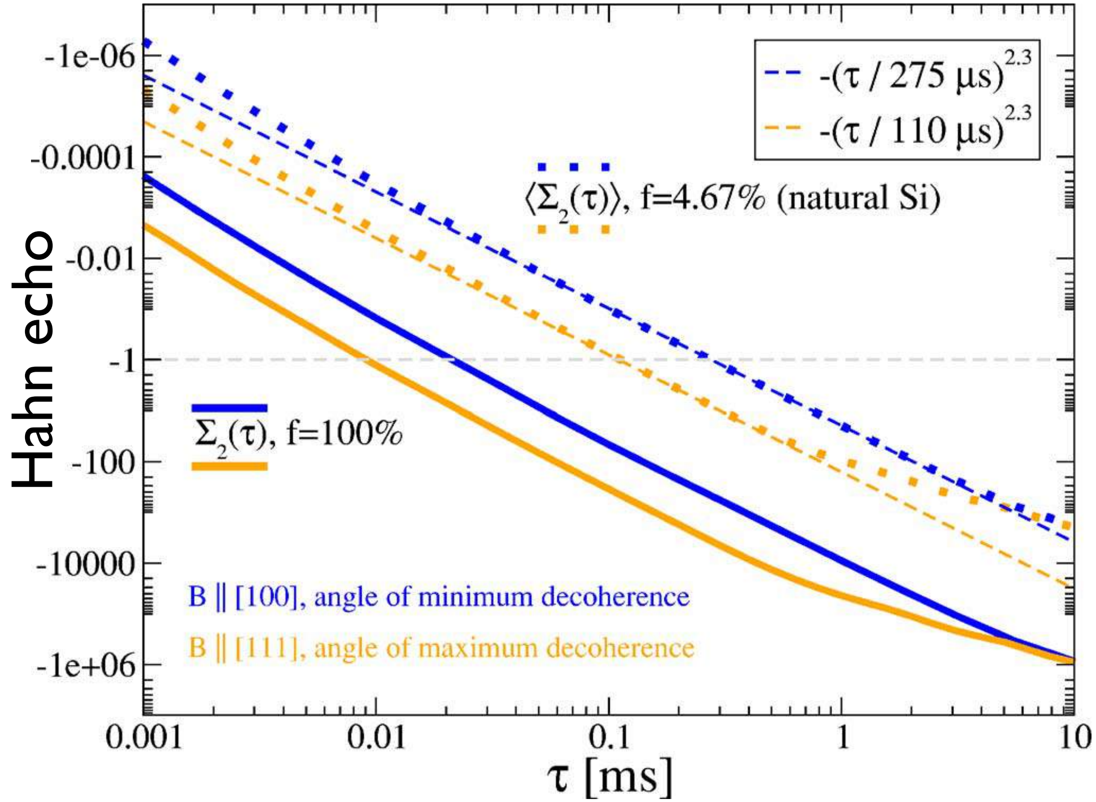
(Received 14 December 2005; revised manuscript received 5 June 2006; published 18 July 2006)





Hahn echo

$$\text{Signal} \propto e^{-(t/T^{HE})^{\alpha}}$$

$$\alpha = 2.3$$

A. M. Tyryshkin and S. A. Lyon (private communication); A. M. Tyryshkin, S. A. Lyon, A. V. Astashkin, and A. M. Raitsimring, Phys. Rev. B **68**, 193207 (2003); .

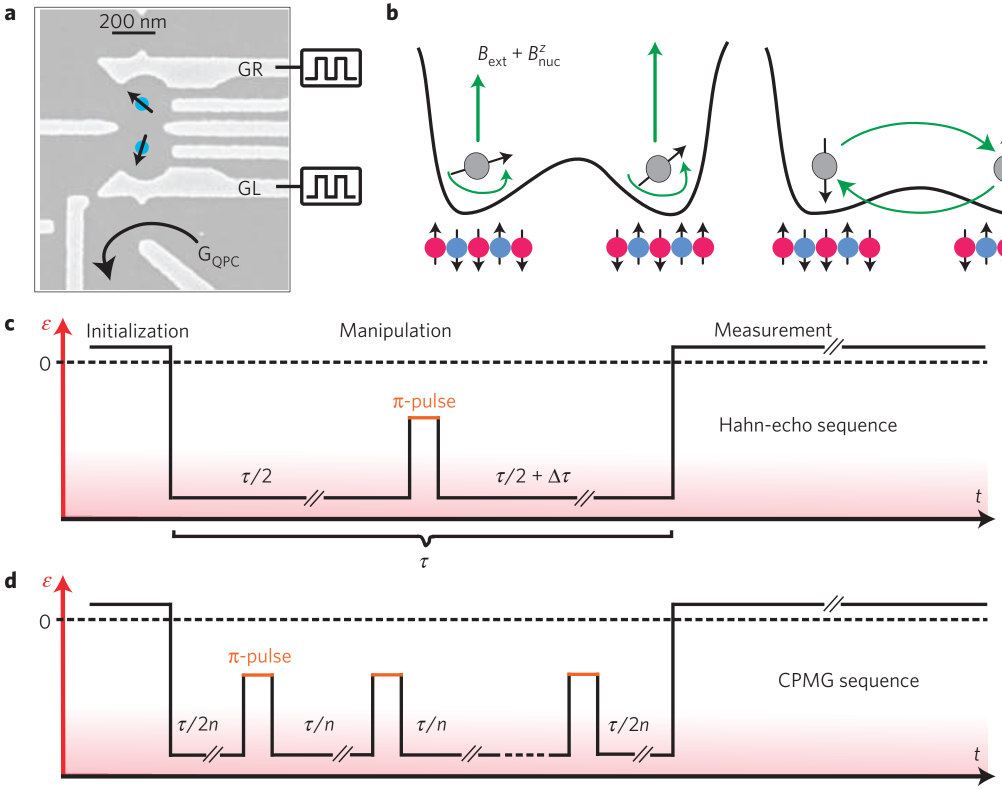
nature physics

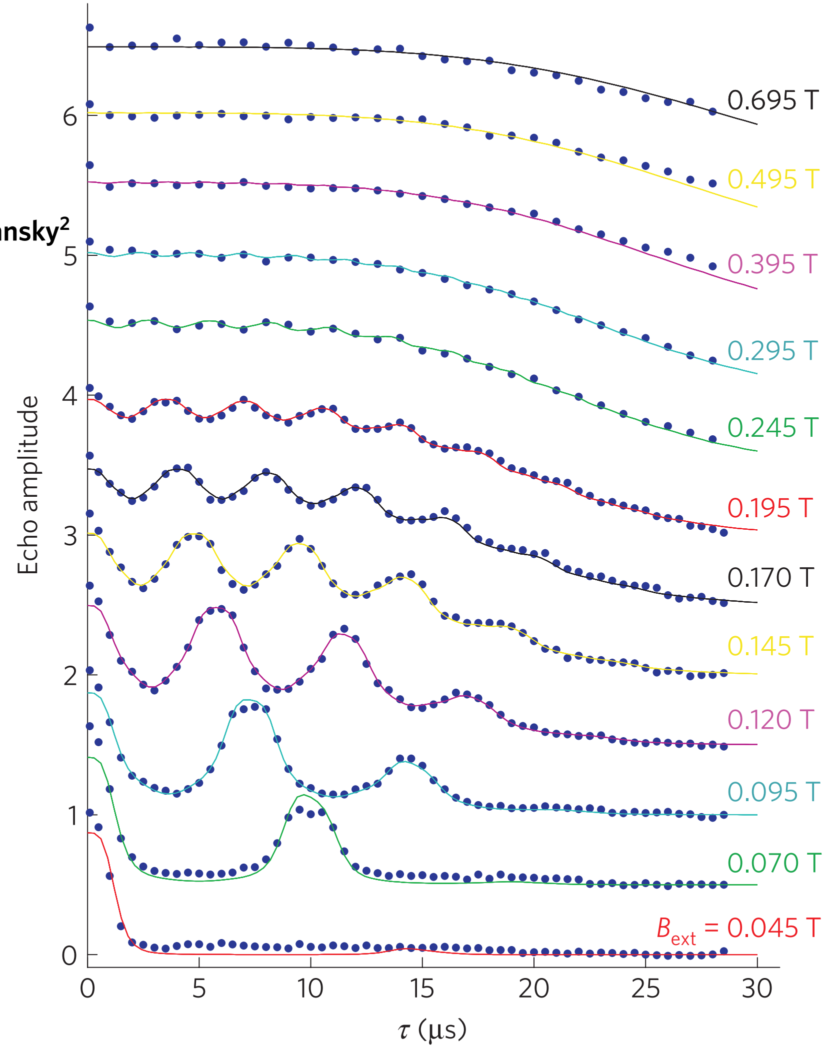
LETTERS

PUBLISHED ONLINE: 12 DECEMBER 2010 | DOI: 10.1038/NPHYS1856

Dephasing time of GaAs electron-spin qubits coupled to a nuclear bath exceeding 200 μs

Hendrik Bluhm[1†], Sandra Foletti[1†], Izhar Neder[1], Mark Rudner[1], Diana Mahalu[2], Vladimir Umansky[2] and Amir Yacoby[1*]





$$\text{Signal} \propto e^{-(t/T^{HE})^{\alpha}}$$

$$\alpha = 4$$

Spin Echo of a Single Electron Spin in a Quantum Dot

F. H. L. Koppens, K. C. Nowack, and L. M. K. Vandersypen

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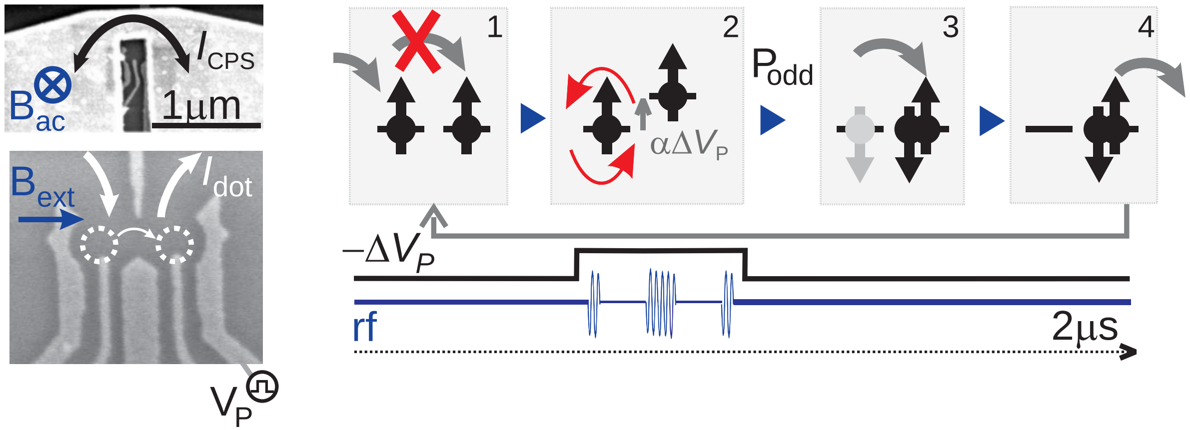
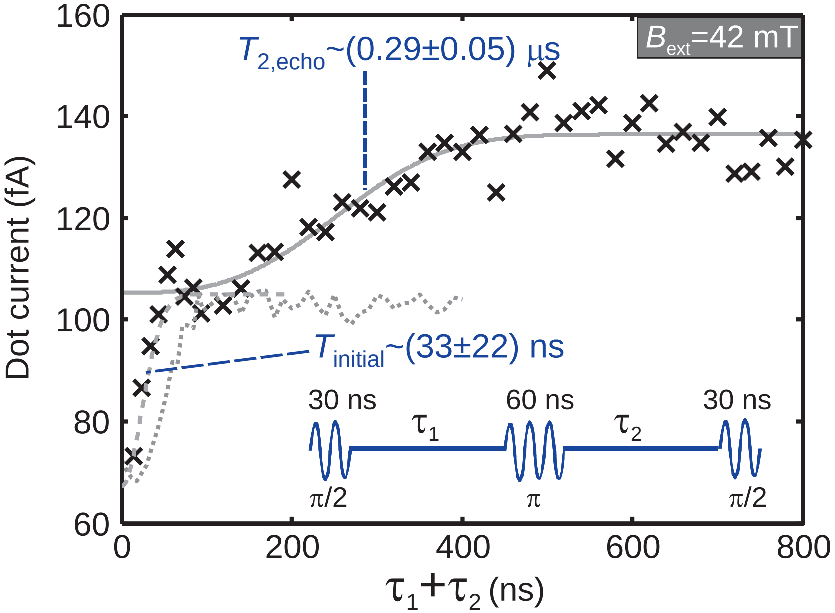
(Received 3 November 2007; published 10 June 2008)



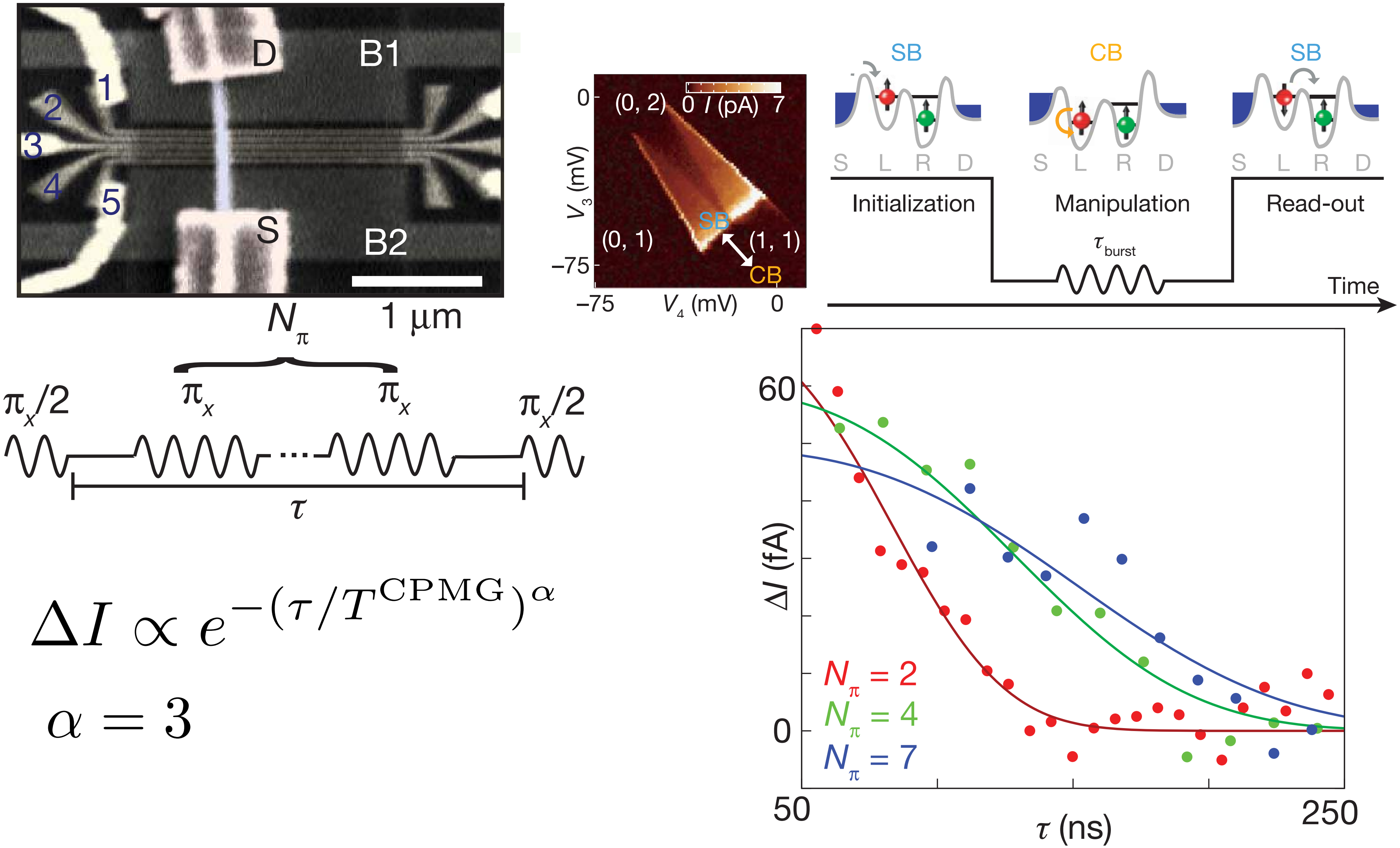


We extract the spin-echo coherence time $T_{2,\mathrm{echo}}$ from a best fit of $a + be^{-[(\tau_1+\tau_2)/T_{2,\mathrm{echo}}]^d}$ to the data (a, b, $T_{2,\mathrm{echo}}$ are fit parameters and d is kept fixed) and find $T_{2,\mathrm{echo}} = (290 \pm 50)$ ns at $B_{\mathrm{ext}} = 42$ mT for $d = 3$ [see Fig. 3(a), solid line]. We note that the precise functional form of the decay is hard to extract from the data, but we find similar decay times and reasonable fits for the range $d = 2$–4.

Spin–orbit qubit in a semiconductor nanowire

S. Nadj-Perge[1]*, S. M. Frolov[1]*, E. P. A. M. Bakkers[1,2] & L. P. Kouwenhoven[1]

| NATURE | VOL 468 | 23/30 DECEMBER 2010



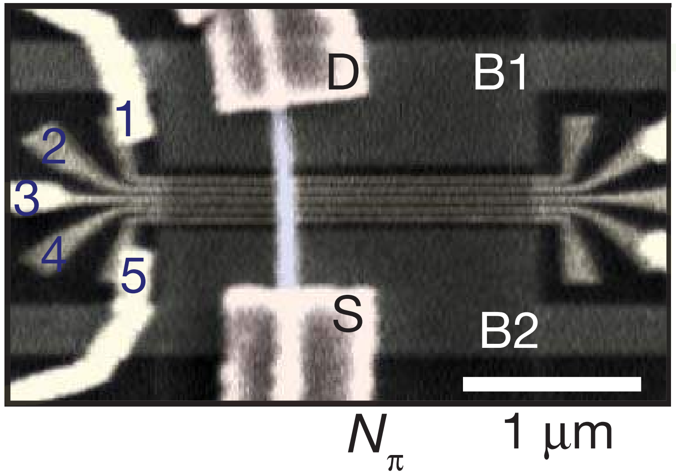
$$\Delta I \propto e^{-(\tau/T^{\mathrm{CPMG}})^{\alpha}}$$

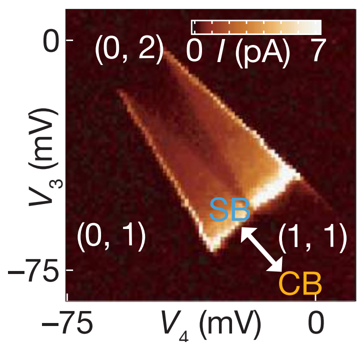
$$\alpha = 3$$

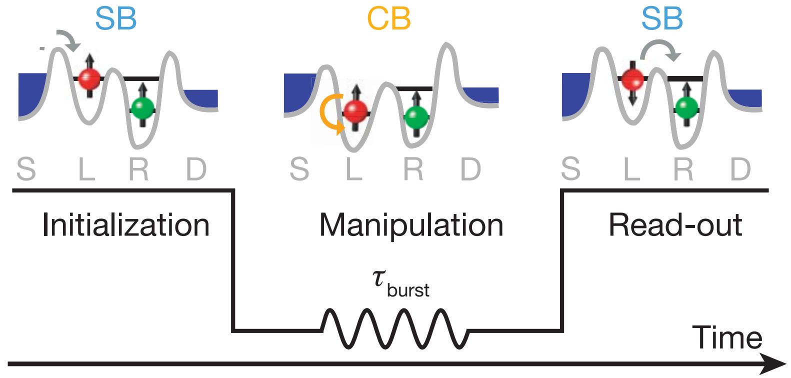
Spin–orbit qubit in a semiconductor nanowire

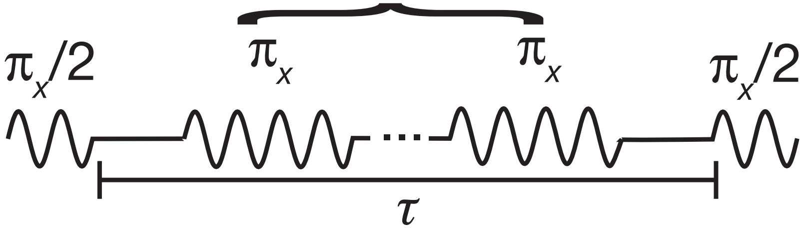
S. Nadj-Perge[1]*, S. M. Frolov[1]*, E. P. A. M. Bakkers[1,2] & L. P. Kouwenhoven[1]

| NATURE | VOL 468 | 23/30 DECEMBER 2010



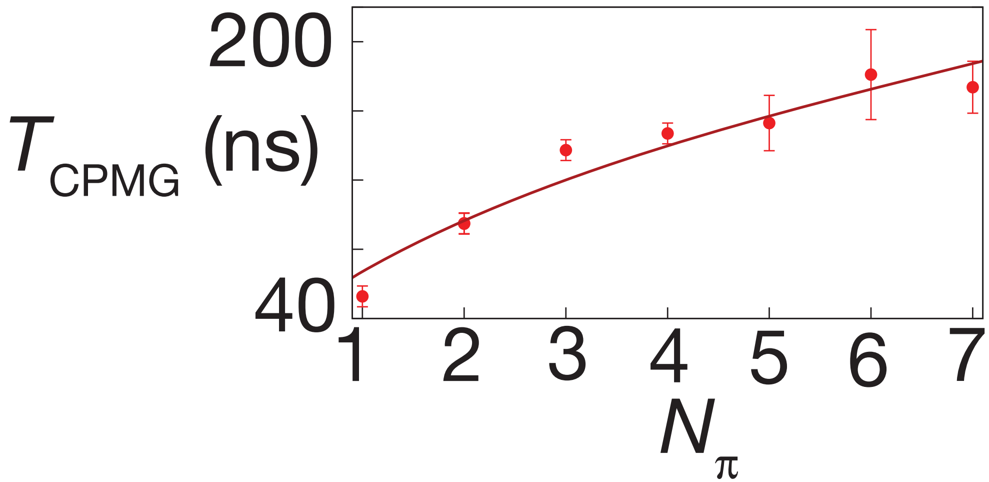






$$T_{\mathrm{CPMG}} \propto (N_\pi)^\gamma$$

$$\gamma = 0.53 \pm 0.1$$



Universal Dynamical Decoupling of a Single Solid-State Spin from a Spin Bath

G. de Lange,[1] Z. H. Wang,[2] D. Ristè,[1] V. V. Dobrovitski,[2] R. Hanson[1*]

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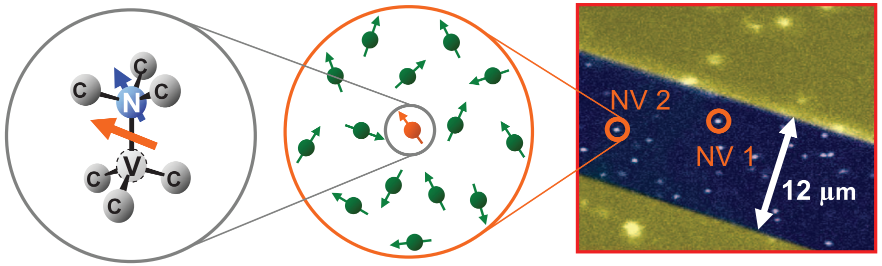
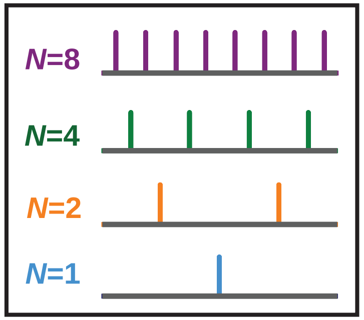
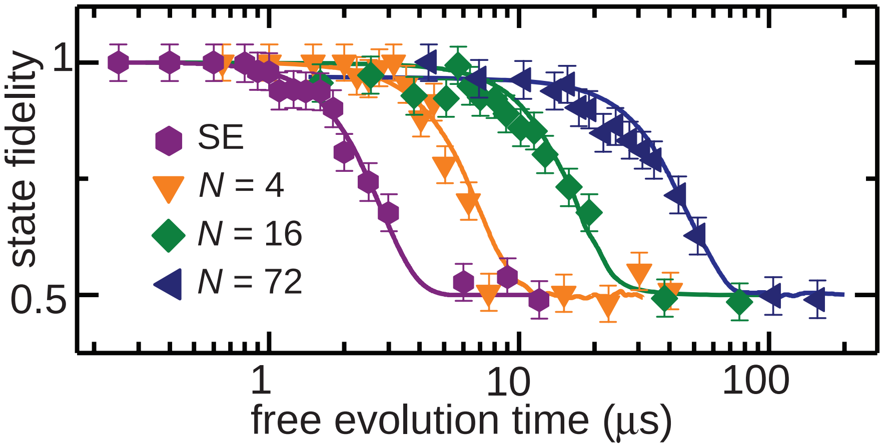
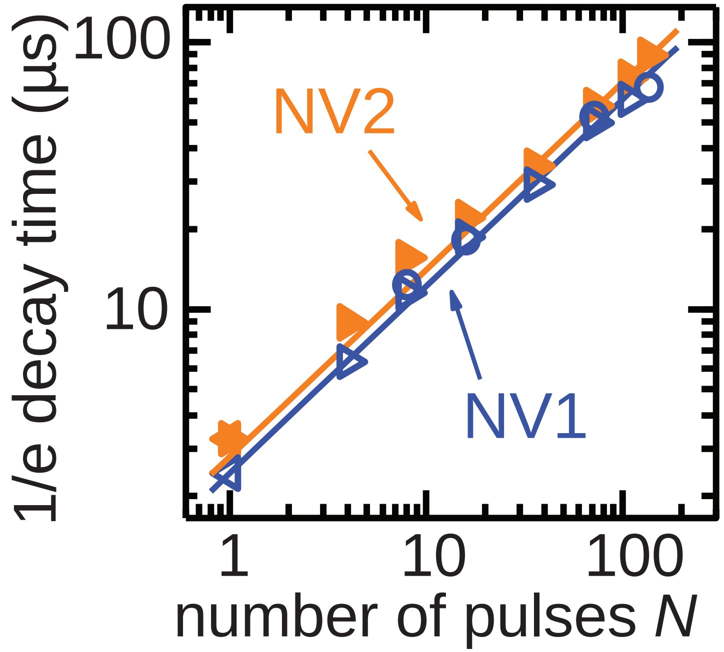








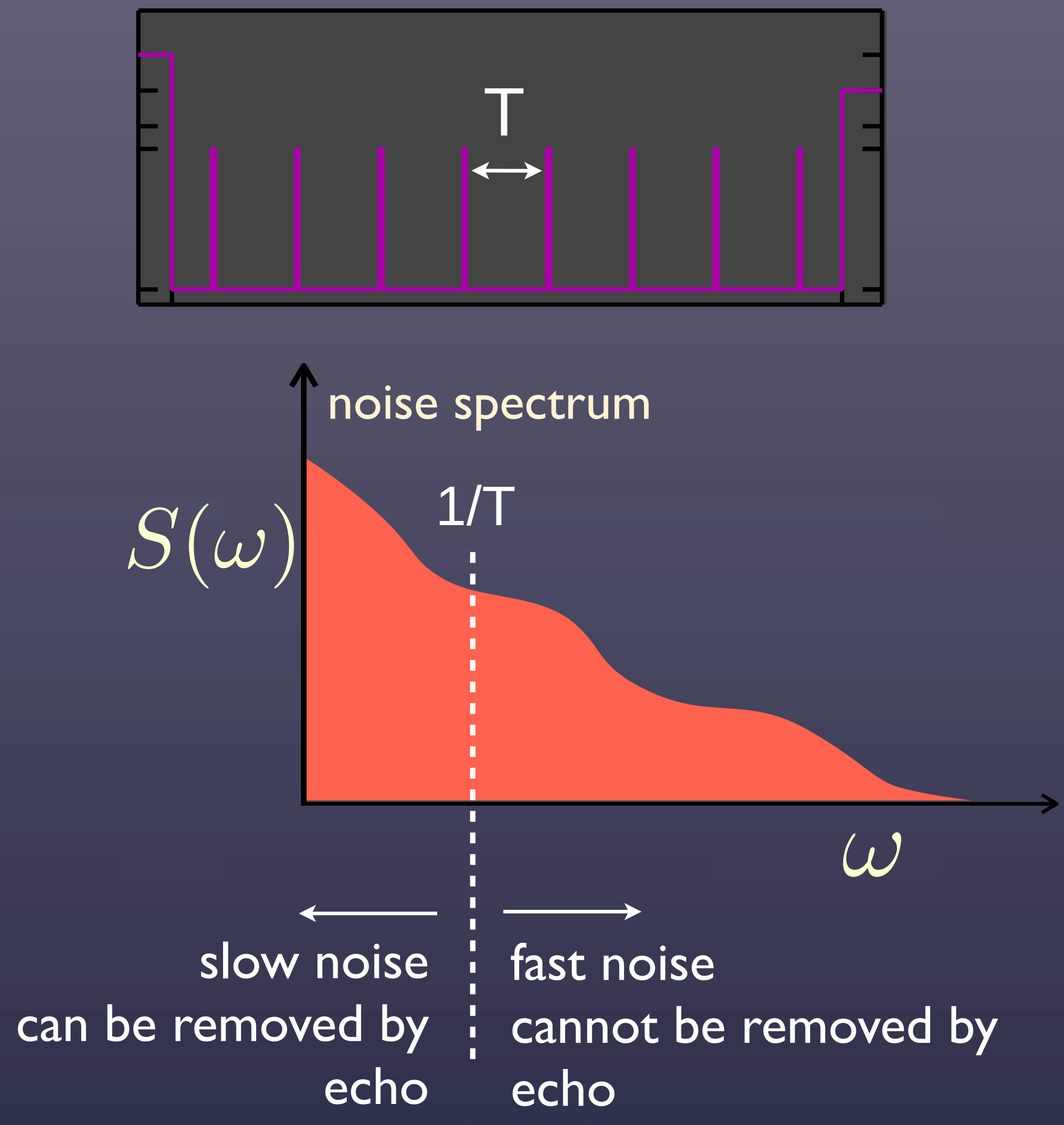
$$e^{-(\tau/T^{\mathrm{CPMG}})^{\alpha}}$$

$$\alpha = 3$$

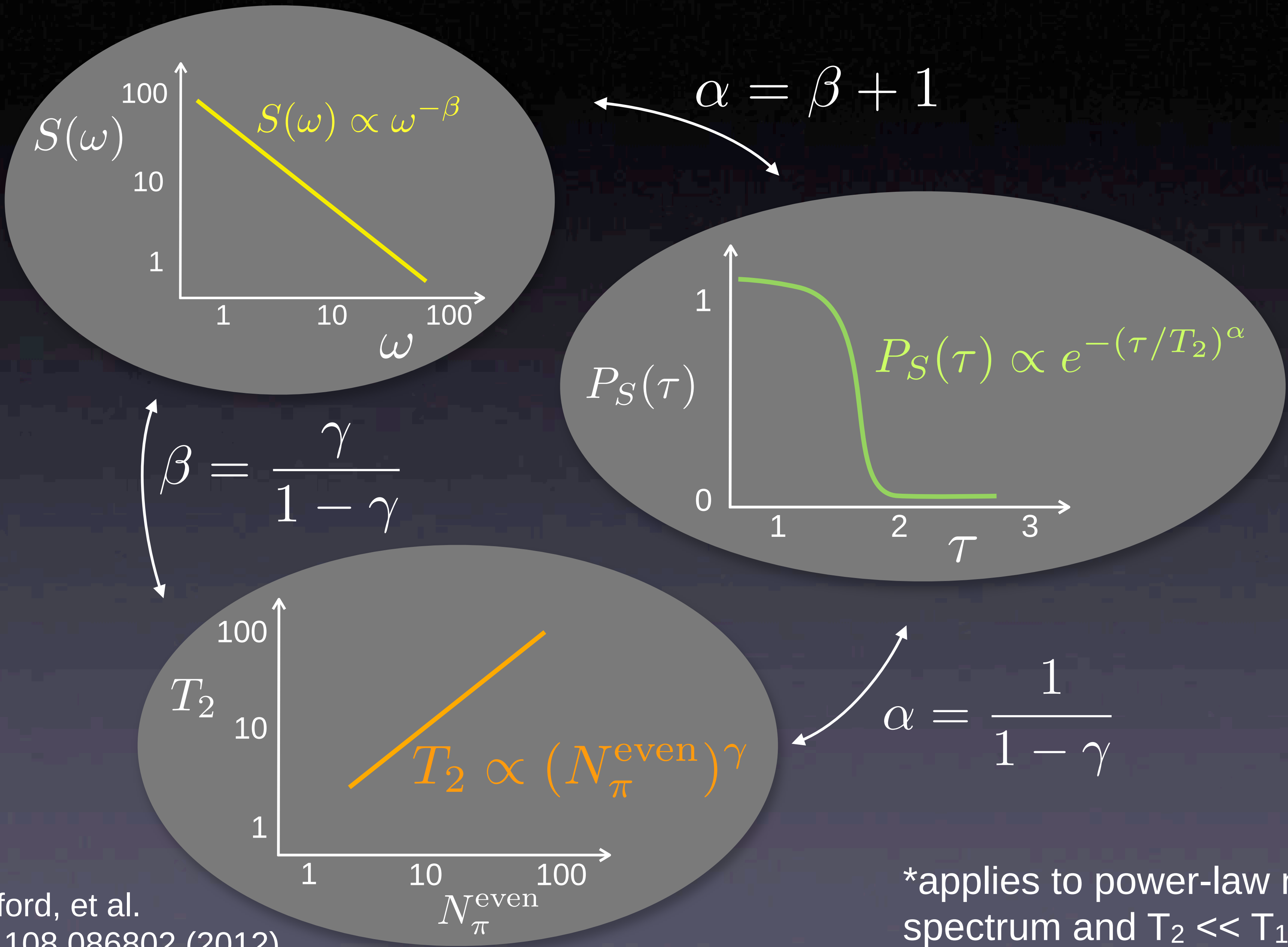
$$T^{\mathrm{CPMT}} \propto N_{\pi}^{\gamma}$$

$$\gamma = 0.66$$

intuition for connection between scaling of dynamical decoupling and noise spectrum



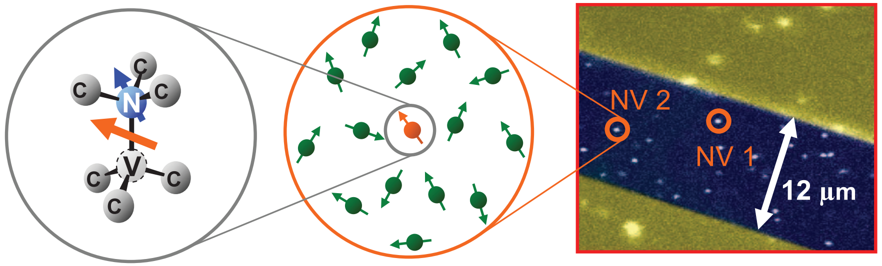
remarkably simple relation between noise spectrum, fidelity envelope, and pulse scaling exponent*



Medford, et al.
PRL 108 086802 (2012)

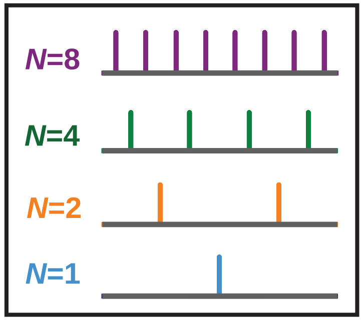
*applies to power-law noise spectrum and $T_2 << T_1$

Universal Dynamical Decoupling of a Single Solid-State Spin from a Spin Bath



G. de Lange,[1] Z. H. Wang,[2] D. Ristè,[1] V. V. Dobrovitski,[2] R. Hanson[1]*

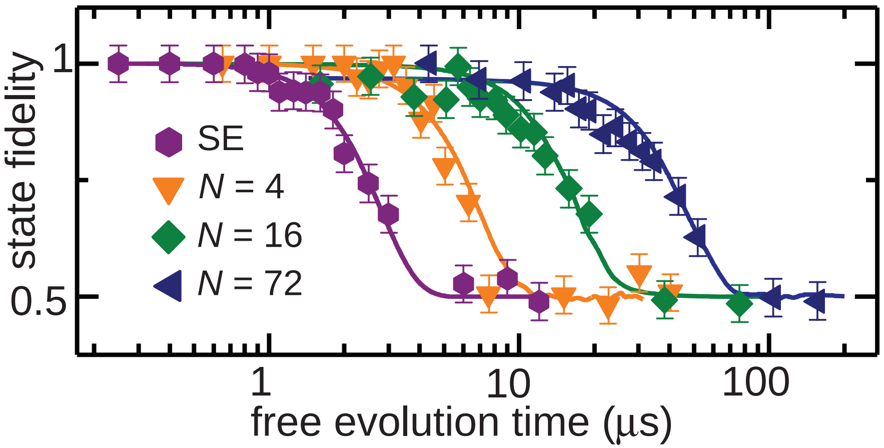
1 OCTOBER 2010 VOL 330 **SCIENCE**



Ornstein-Uhlenbeck

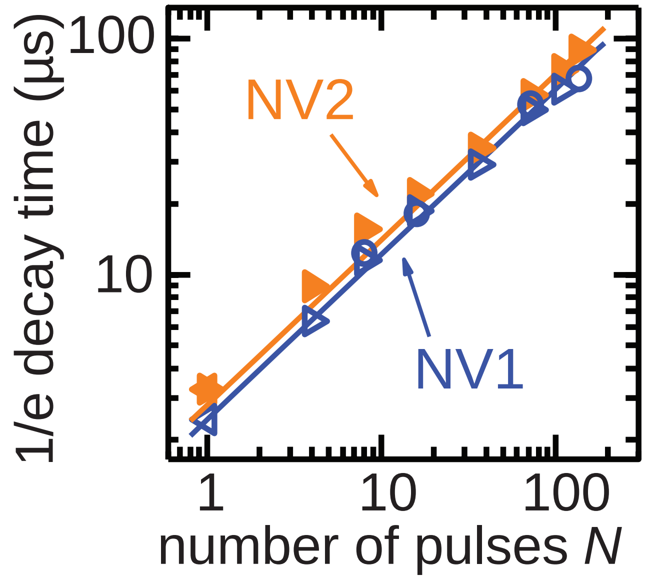
$$S(\omega) \propto \omega^{-\beta}$$

$$\beta = 2$$



$$e^{-(\tau/T^{\mathrm{CPMG}})^{\alpha}}$$

$$\alpha = \beta + 1 = 3$$



$$T_2 \propto N^{\gamma}$$

$$\gamma = \frac{\beta}{\beta + 1} = 2/3$$

Scaling of Dynamical Decoupling for Spin Qubits

J. Medford,[1] Ł. Cywiński,[2] C. Barthel,[1] C. M. Marcus,[1] M. P. Hanson,[3] and A. C. Gossard[3]

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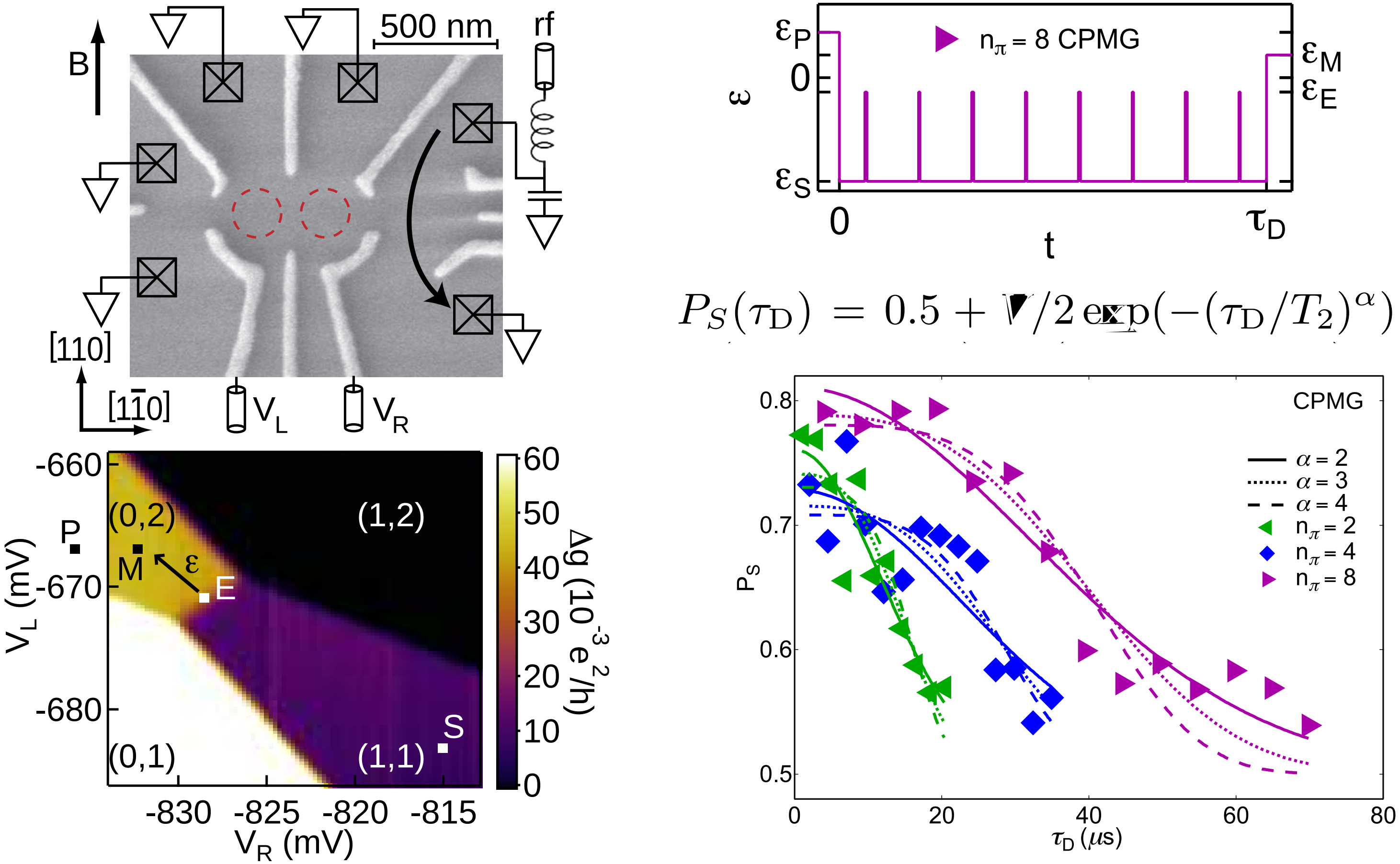
(Received 18 August 2011; published 23 February 2012)



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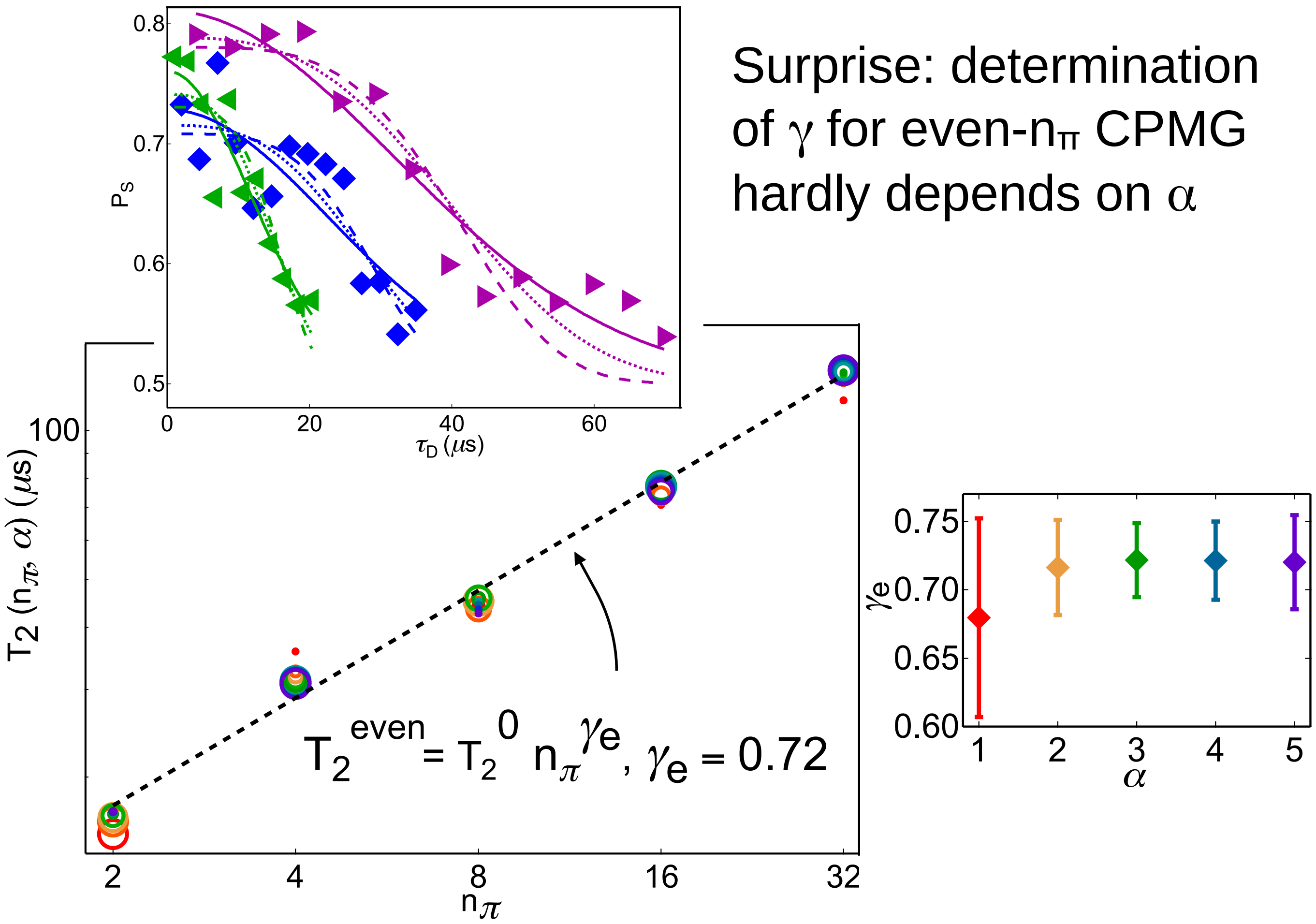
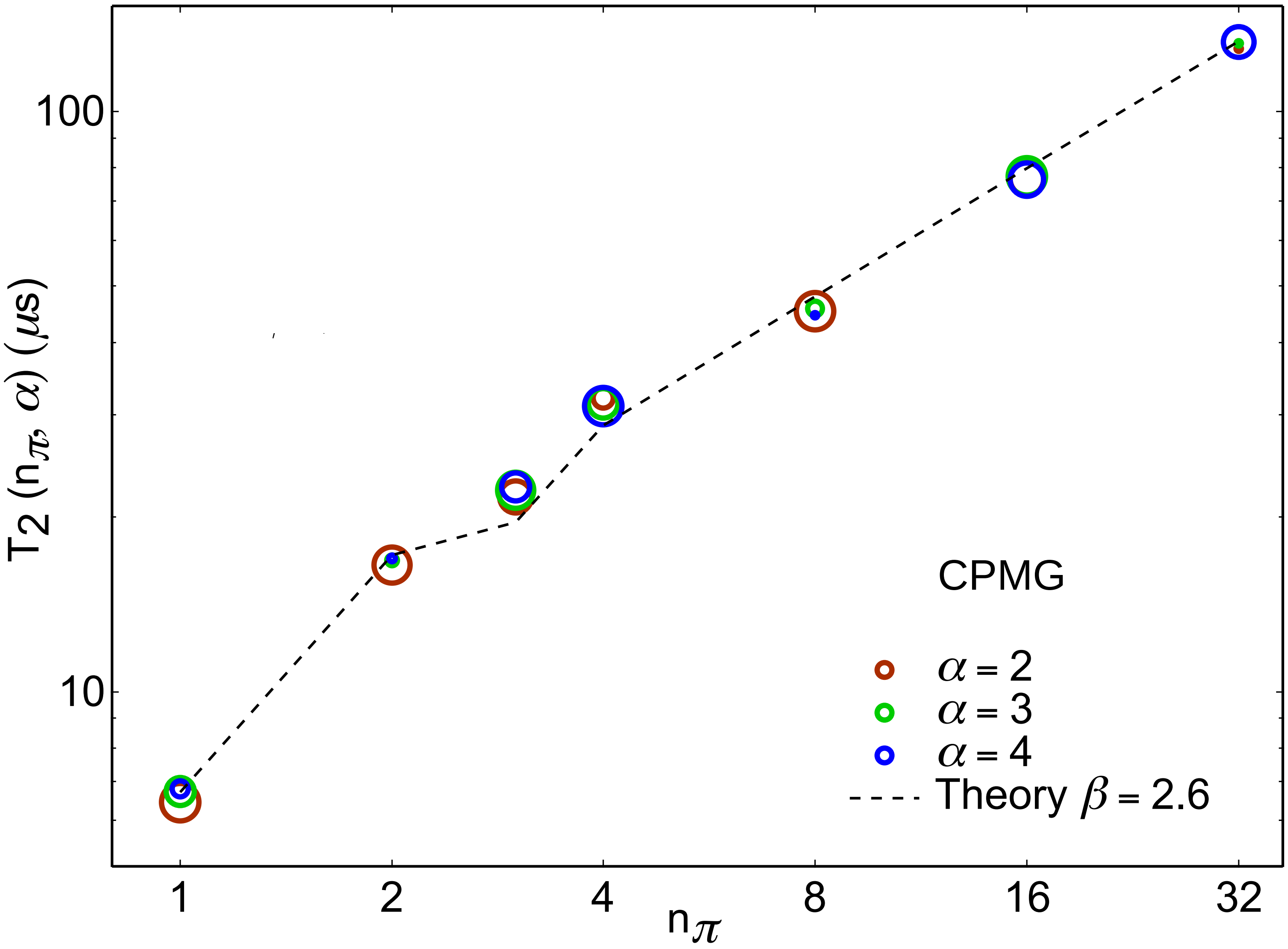
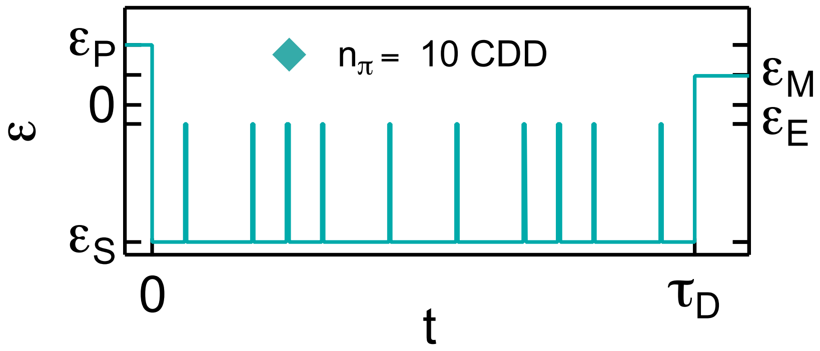


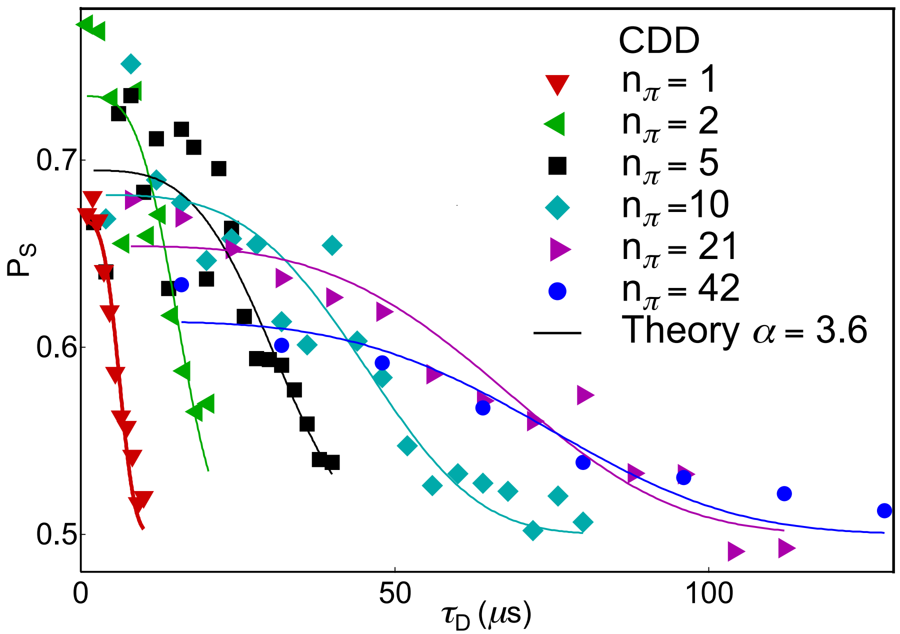
Surprise: determination of γ for even-n_π CPMG hardly depends on α

full numerical solution for even and odd n_π



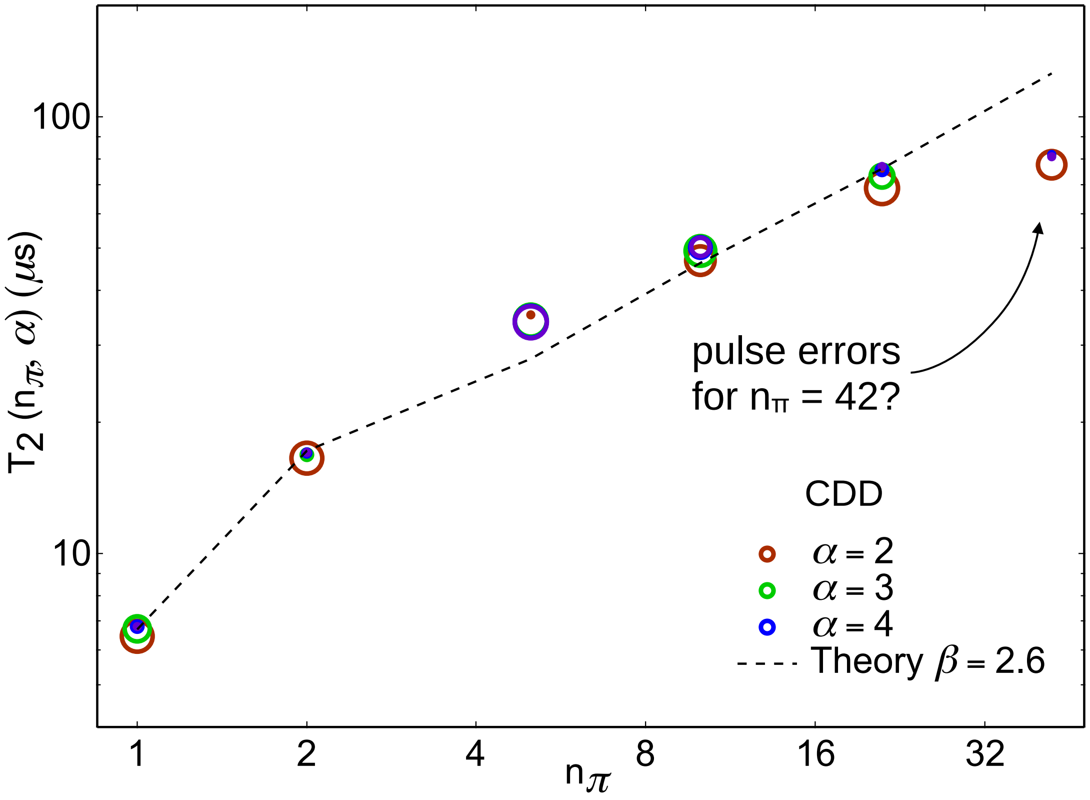
Full numerical solution for Concatinated Dynamical Decouping (CDD)



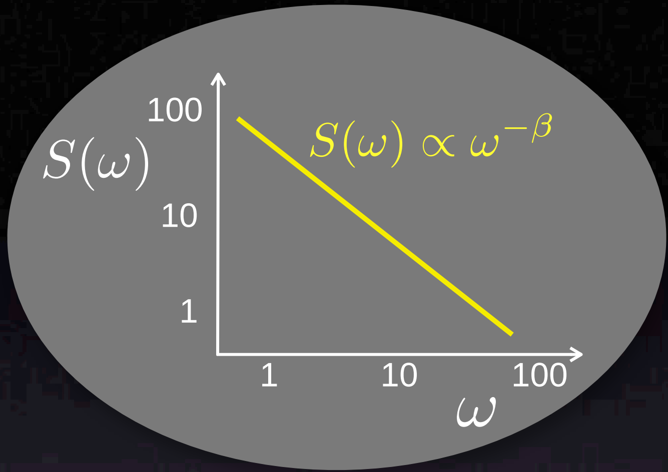


$$S(\omega) = A^{\beta+1}/\omega^{\beta}$$

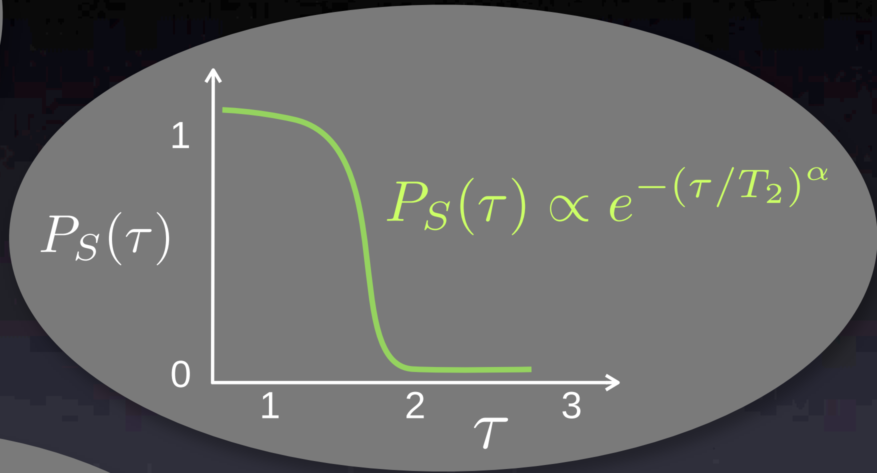
A and β are fit parameters based on even-n_π CPMG.



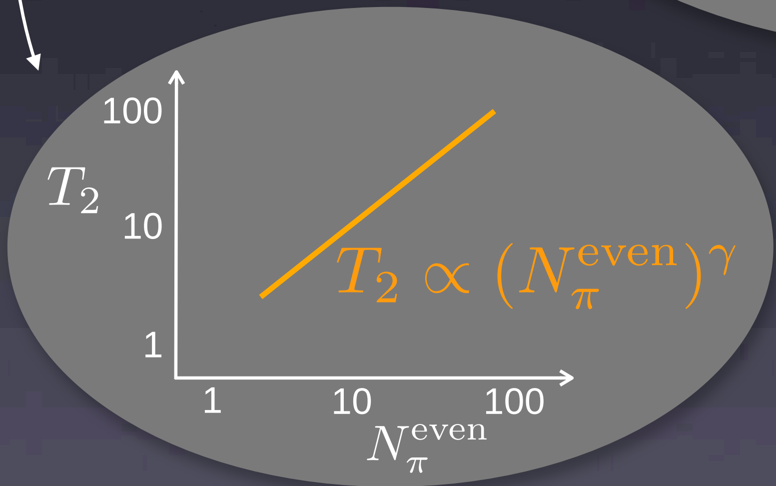
Values for GaAs DQD



$$\alpha = \beta + 1$$



$$\beta = \frac{\gamma}{1-\gamma}$$



$$\alpha = \frac{1}{1-\gamma}$$

$$\gamma = 0.72 \pm 0.01$$

$$\beta = \frac{\gamma}{1-\gamma} = 2.6 \pm 0.1$$

$$\alpha = \beta + 1 = 3.6 \pm 0.1$$

Medford, et al.
PRL 108 086802 (2012)

PHYSICAL REVIEW B **84**, 075339 (2011)

Nuclear spin dynamics in double quantum dots: Fixed points, transients, and intermittency

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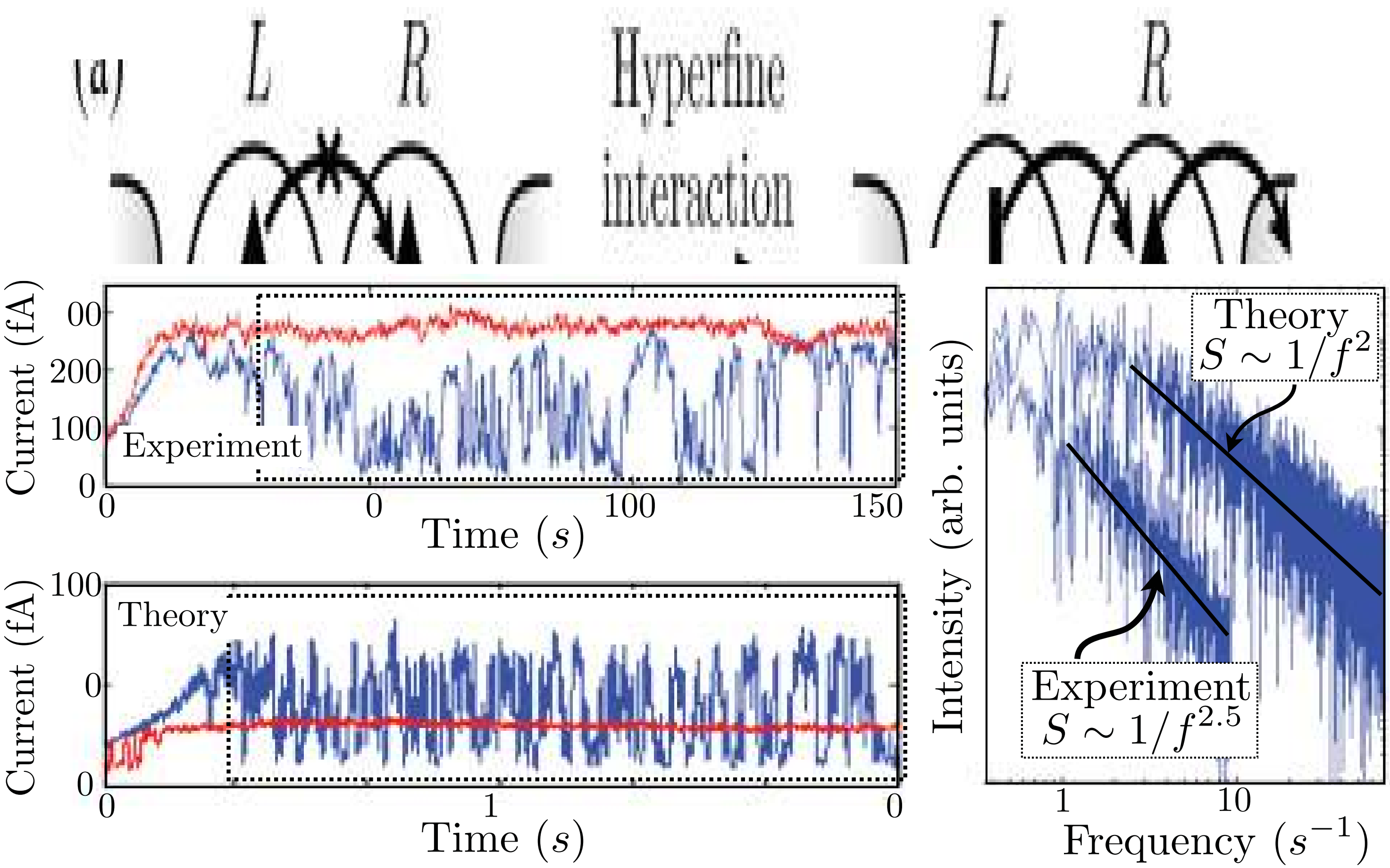
(Received 8 July 2011; published 17 August 2011)



Storing quantum information for 30 seconds in a nanoelectronic device

Juha T. Muhonen,[1] Juan P. Dehollain,[1] Arne Laucht,[1] Fay E. Hudson,[1] Takeharu Sekiguchi,[2] Kohei M. Itoh,[2] David N. Jamieson,[3] Jeffrey C. McCallum,[3] Andrew S. Dzurak,[1] and Andrea Morello[1]

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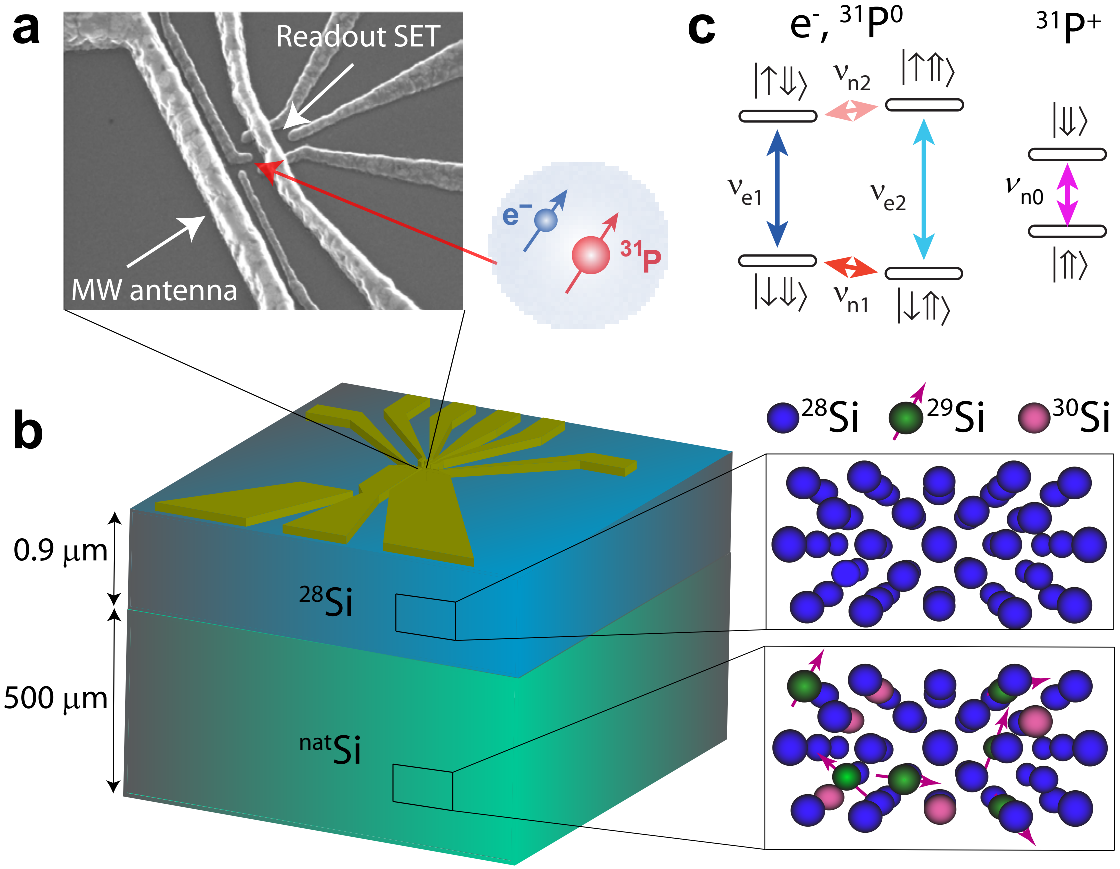

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